

Texas A & M University
Department of Mechanical Engineering
MEEN 651 Control System Design

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**Lecture 2B: Sampling and Aliasing Phenomena,
Sampling Rate Selection and Nyquist Frequency**

The objective of this lecture is to introduce and explain some of the most basic concepts that are involved in the measurement of continuous-time signals. The basic rules regarding sampling are discussed and the consequences of incorrect sampling rate selection are mentioned.

Definition:

The instantaneous values of a signal at equally or unequally spaced values of time are called *samples*.

Sampling, if done correctly, can help us represent a continuous-time signal very accurately. The correct way of sampling a continuous-time signal is described by the so-called *sampling theorem*.

Sampling of Continuous-Time Signals

In general, if we were to sample a signal we would not expect to be able to guess the precise continuous-time signal that generated the samples. In fact, there are infinite number of signals that pass through a finite number of points, as shown in Figure 1.

Strangely enough, for most signals, if we take the samples sufficiently close to each other we can uniquely identify (or reconstruct) the continuous-time signal that generated the samples. The above statement forms the basis for the accurate measurement of dynamic signals, such as vibration level.

Impulse-Train Sampling

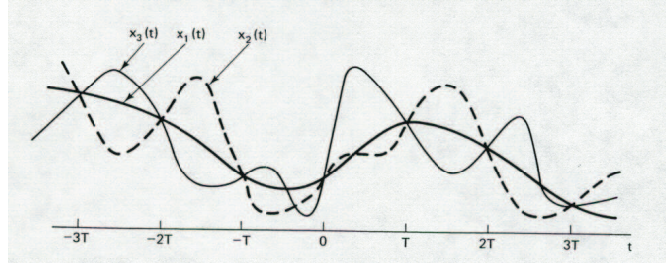


Figure 1: Three Continuous Time Signals with Identical Values at Integer Multiples of T .

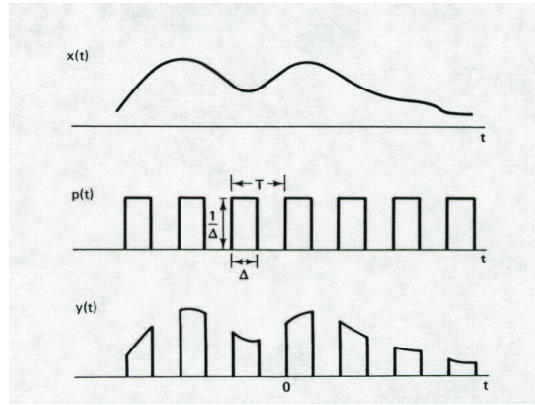


Figure 2: Pulse Amplitude Modulation. As $\Delta \rightarrow 0$, the $p(t)$ Approaches an Impulse.

In general, a simple way to sample a continuous-time signal is to measure it for a finite period of time using pulses, as shown in Figure 2. As the duration of the pulses approaches zero, we obtain the impulse train shown in Figure 3, for which the individual impulses have values corresponding to instantaneous samples of the continuous-time signal at time instants spaced T seconds apart.

The problem is that if we arbitrarily space the impulses shown in Figure 3, then we will not be able to uniquely know the signal that generated the samples. However, if the impulses are appropriately spaced, that is if we sample the continuous-time signal with a predetermined sampling rate, then we can uniquely determine its nature in the continuous-time domain. The determination of the sampling rate needed for a signal is computed from the *sampling theorem* using the formula

$$\omega_s > 2\omega_M, \quad (1)$$

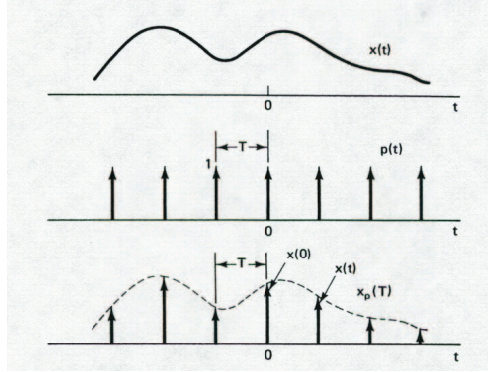


Figure 3: Pulse Amplitude Modulation with an Impulse Train.

where ω_M is the maximum frequency contained in the signal and ω_s is the sampling rate.

Equation (1) says that for accurate representation in the discrete-time domain, a continuous-time signal must be sampled at least twice the maximum frequency present in that signal. That is, the sampling frequency must be sufficiently high, so that the spacing of the impulses shown in Figure 3 is close enough to give us an accurate picture of the continuous-time signal being sampled. The sampling frequency ω_s is called *Nyquist frequency*. The frequency $2\omega_M$ which must be exceeded is called *Nyquist rate*.

The Effect of Undersampling: Aliasing

In our previous discussion we argued that if in our measurements the sampling rate used is high enough, we can identify or reconstruct a continuous-time signal uniquely. If for some reason our sampling rate is not high enough, that is if we undersample a signal, then we end-up with an effect called *aliasing*. This phenomenon results in obtaining very inaccurate and false information regarding the signal being sampled.

A simple explanation for the reasons behind this phenomenon is given in Figures 4 and 5. In this figure a sinusoidal signal with increasing frequency is sampled using impulse trains of constant spacing. In part (a) of the figure the sampling rate is six times the frequency of the sinusoid, i.e. six samples are collected during each complete cycle. Since the sampling rate is more than twice the frequency of the sinusoid, the signal can be reconstructed. In part (b) the sampled signal is twice the frequency of the signal in part (a), whereas the sampling rate is kept the same. As a result, the sampling rate is three times the frequency of the

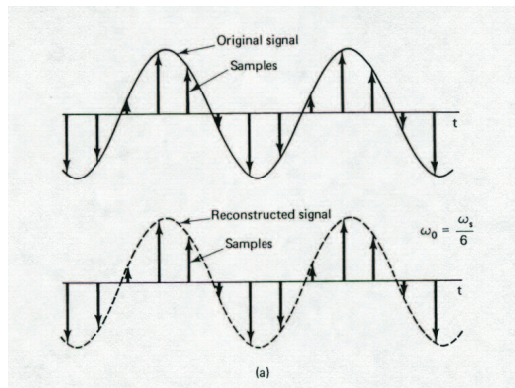


Figure 4: Effects of Aliasing on a Sinusoidal Signal.

sinusoid and, again, the signal can be uniquely reconstructed¹. In part (c) of the figure the sinusoidal signal frequency is increased to four times the frequency of the signal in part (a). By keeping the sampling rate the same as in part (a) we have created a serious problem. The sampling rate is now 1.5 times the signal frequency (less than the 2 required by the sampling theorem). As a result we are not able to uniquely reconstruct the sinusoid shown in part (c) of the figure². Any further increase in the frequency of the sinusoidal signal without appropriate increases in the sampling rate results in similar *aliasing*. This is shown in part (d) of the figure also³.

Reading Assignment

Read the examples Handout E.4 posted on the course web page.

¹Note that there are three samples collected per cycle, including the sample at the zero-crossing points.

²Note that there are three samples every two cycles.

³Note that there are six samples every five cycles.

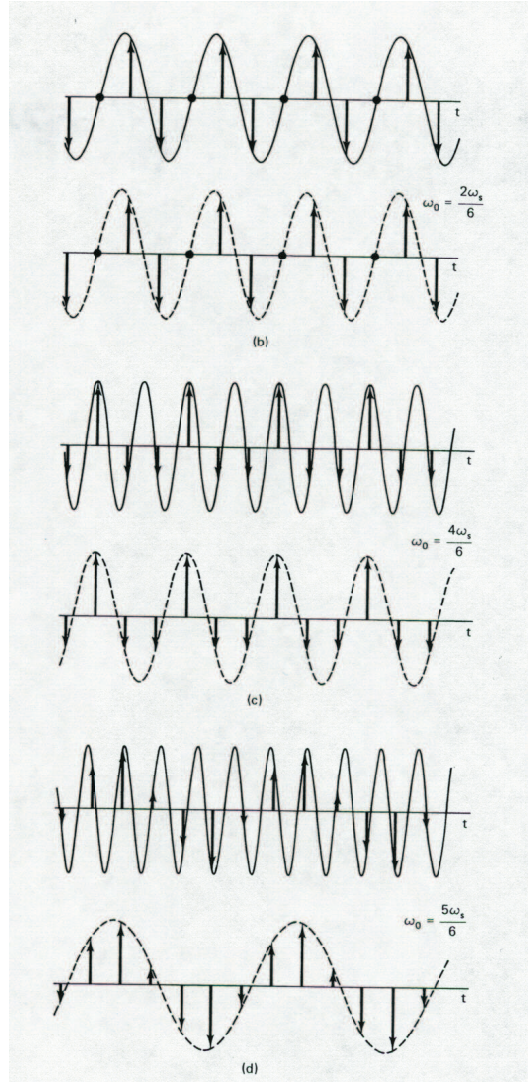


Figure 5: Effects of Aliasing on a Sinusoidal Signal (continued).