

EEM 214

Lecture 5

BJT Transistors

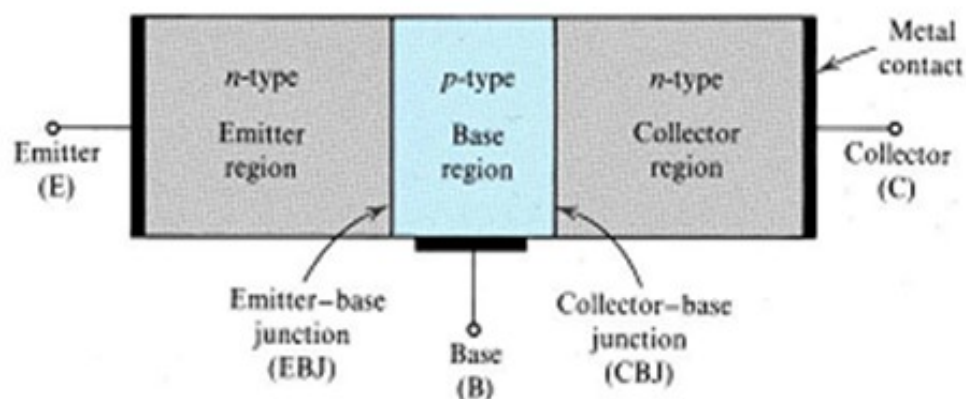
Bipolar Junction Transistor (BJT)

- A three-terminal device that uses the voltage of the two terminals to control the current flowing in the third terminal.
 - The basis for amplifier design.
 - The basis for switch design.
 - The basic element of high speed integrated digital and analog circuits.
- Applications
 - Discrete-circuit design.
 - Analog circuits.
 - * High frequency application such as radio frequency analog circuit.
 - Digital circuits.
 - * High speed digital circuit such as emitter coupled circuit (ECC).
 - * Bi-CMOS (Bipolar+CMOS) circuits that combines the advantages of MOS-FET and bipolar transistors.
 - MOSFET: high-input impedance and low-power.
 - Bipolar transistors: high-frequency-operation and high-current-driving capabilities.

Structure

NPN Transistor

- Emitter (E): heavily doped n-type region.
- Base (B): lightly doped p-type region.
- Collector (C): heavily doped n-type region.
- Two diodes connected in series with opposite directions.
 - * EBJ: Emitter-Base junction.
 - * CBJ: Collector-Base junction.



A simplified structure of the NPN transistor.

There are four basic modes of operation for a BJT depending on the states of the two *pn* junctions of the transistor:

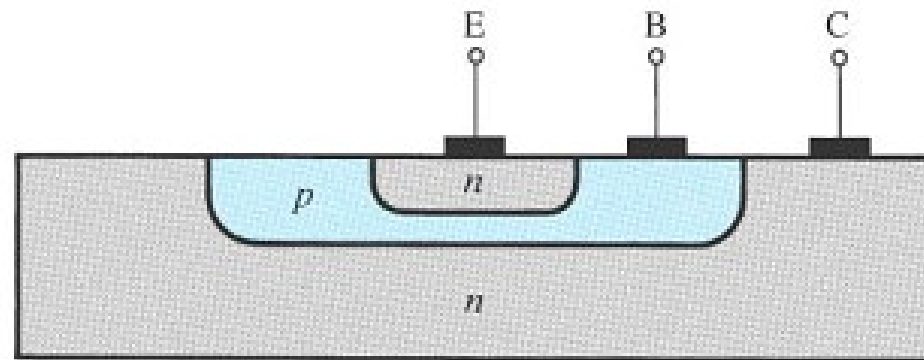
Mode	Emitter-Base Jct.	Collector-Base Jct.
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward
Reverse Active	Reverse	Forward

In digital logic applications, the transistor switches between the cutoff and saturation modes. As a linear amplifier in a communication circuit, the transistor would operate in the active mode.

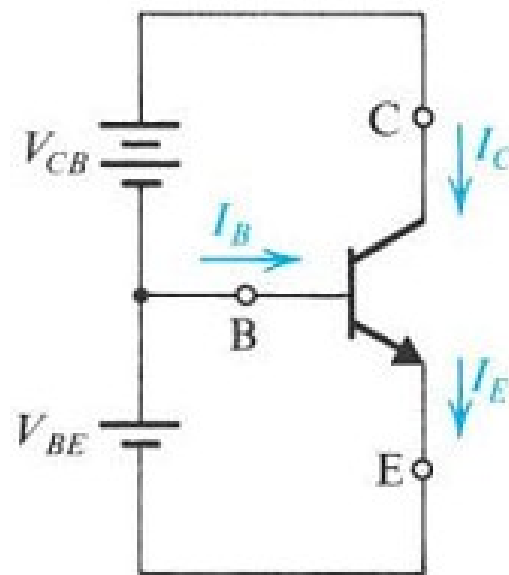
Because of the asymmetrical physical construction, the reverse active mode is not the same as interchanging the collector and emitter leads.

The states of the two *pn* junctions can be altered by the external circuitry connected to the transistor. This is called **biasing** the transistor.

- Cutoff
 - * EBJ (Reverse), CBJ (Reverse)
 - * $v_{BE} < 0, v_{CB} > 0$.
- Active (refer to Figure 7.7)
 - * EBJ (Forward), CBJ (Reverse)
 - * $v_{BE} > 0, v_{CB} > 0$.
- Reverse Active
 - * EBJ (Reverse), CBJ (Forward)
 - * $v_{BE} < 0, v_{CB} < 0$.
- Saturation
 - * EBJ (Forward), CBJ (Forward)
 - * $v_{BE} < 0, v_{CB} < 0$.

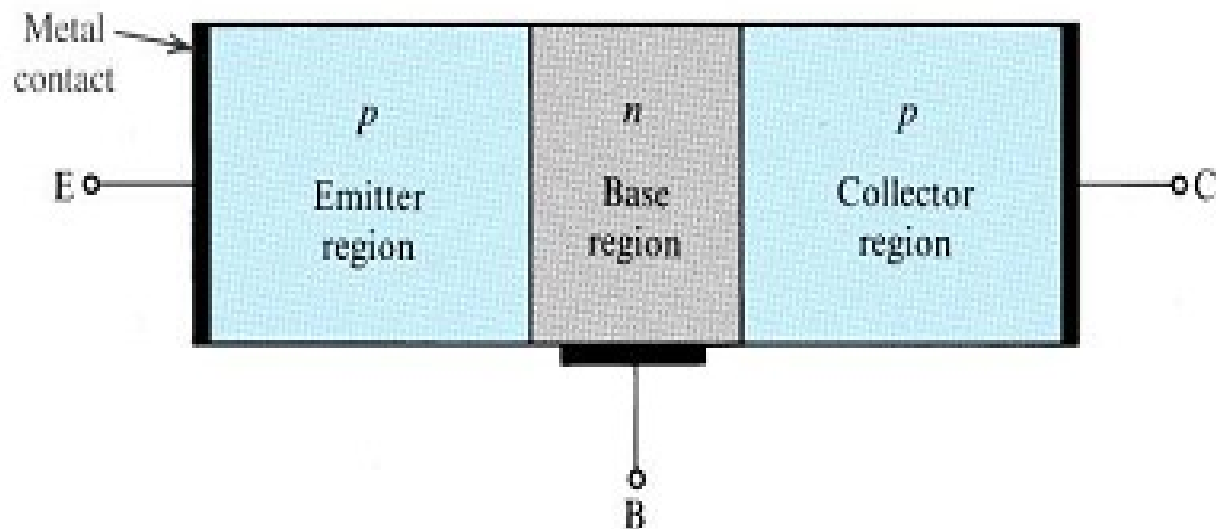


Cross-section of an NPN BJT.



Voltage polarities and current flow in the NPN transistor biased in the active mode.

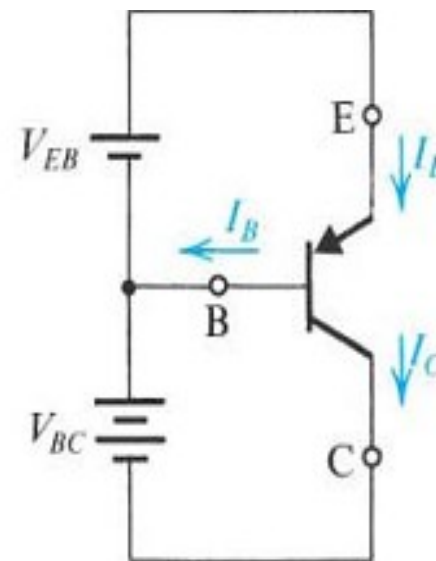
PNP Transistor



A simplified structure of the PNP transistor.

- Emitter (E): heavily doped p-type region.
- Base (B): lightly doped n-type region.
- Collector (C): heavily doped p-type region.
- Two diodes connected in series with opposite directions.
 - * EBJ: Emitter-Base junction.
 - * CBJ: Collector-Base junction.

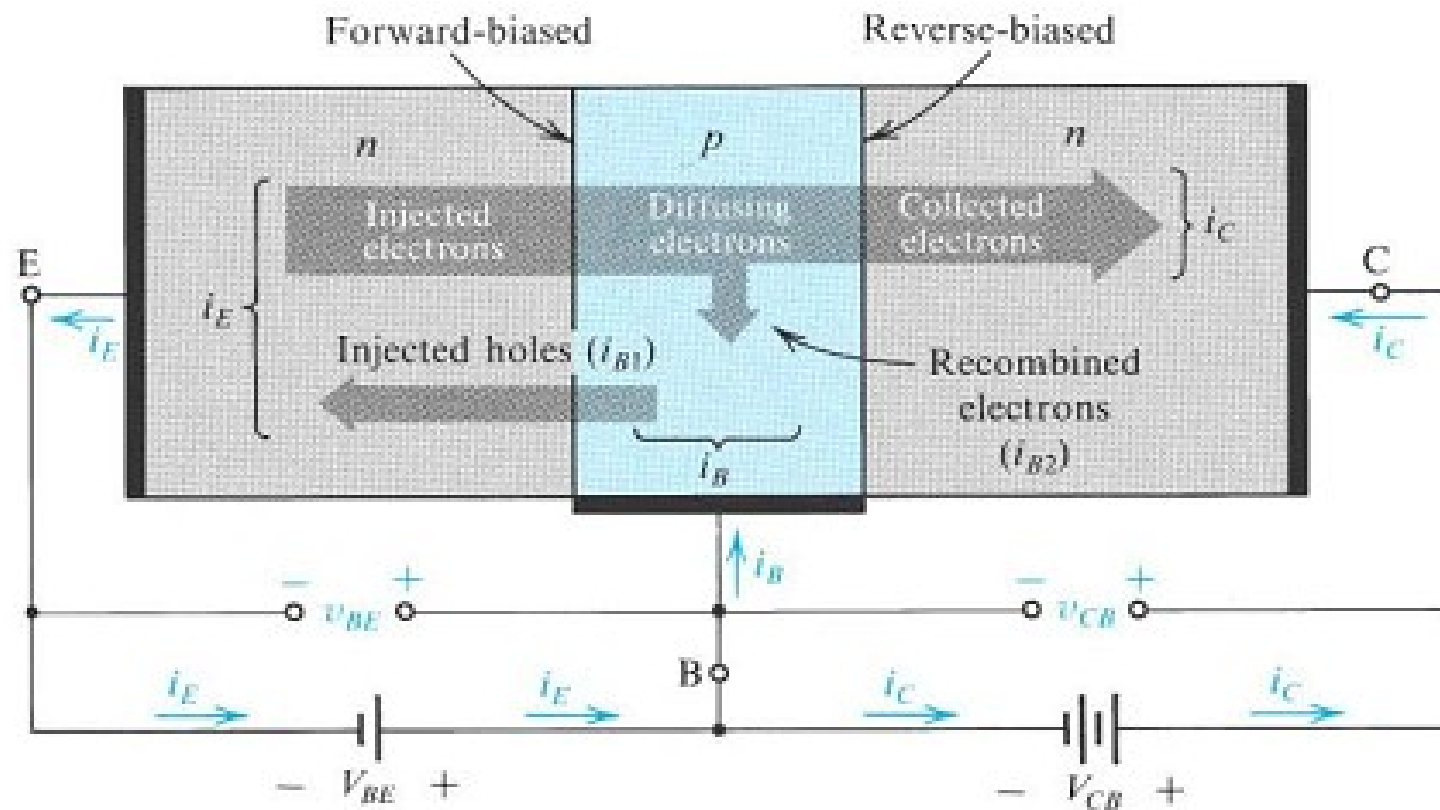
- Modes of operations
 - Cutoff
 - * EBJ (Reverse), CBJ (Reverse)
 - * $v_{EB} < 0, v_{BC} < 0$.
 - Active (refer to Figure 7.7)
 - * EBJ (Forward), CBJ (Reverse)
 - * $v_{EB} > 0, v_{BC} > 0$.
 - Reverse Active
 - * EBJ (Reverse), CBJ (Forward)
 - * $v_{EB} < 0, v_{BC} < 0$.
 - Saturation
 - * EBJ (Forward), CBJ (Forward)
 - * $v_{EB} > 0, v_{CB} > 0$.



Voltage polarities and current flow in the PNP transistor biased in the active mode.

Operations of NPN Transistor

Active Mode



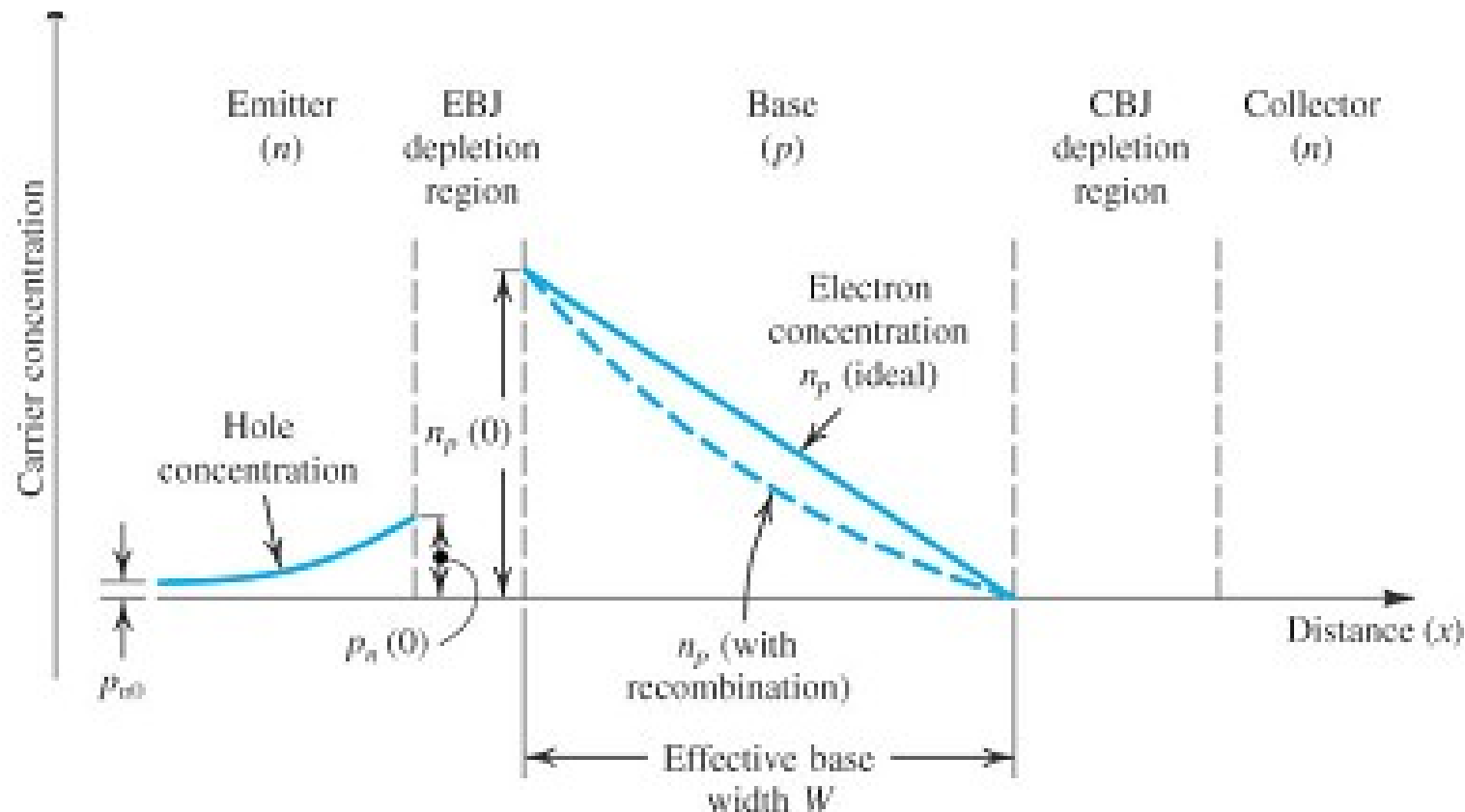
Current flow in an NPN transistor to operate in the active mode.

- Emitter-Base Junction

- Forward bias, $v_{BE} > 0$.
- Electrons in the emitter region are injected into the base causing a current i_{E1} .
- Holes in the base region are injected into the emitter region causing a current i_{E2} .

* Generally, $i_{E1} \gg i_{E2}$.

$$i_E(t) = i_{E1} + i_{E2}$$



Profiles of minority carrier concentrations in the base and in the emitter of an NPN transistor.

- Base region
 - Tapered concentration causes the electrons to diffuse through the base region toward the collector.
 - * Some of the electrons may combine with the holes causing a concave shape of the profile.
 - * The recombination process is quite small due to lightly doped and thin base region.

$$n_p(0) = n_{p0} e^{v_{BE}/V_T}$$

- Diffusion current I_n (flowing from right to the left) is proportional to the slope of the concentration profile.
 - * A_E is the cross-sectional area of the base-emitter junction.
 - * D_n is the electron diffusivity in the base region.
 - * W is the effective width of the base.

$$I_n = A_E q D_n \frac{dn_p(x)}{dx} = -A_E q D_n \frac{n_p(0)}{W}$$

- Collector-Base Junction
 - Reverse bias, $v_{BC} > 0$.
 - The electrons near the collector side are swept into the collector region causing zero concentration at the collector side.

- Collector current, i_C .
 - Most of the diffusing electrons will reach the collector region, i.e., $i_C = -I_n$.
 - * Only a very small percentage of electrons are recombined with the holes in the base region.
 - As long as $v_{CB} > 0$, i_C is independent of v_{CB} .
 - * The electrons that reach the collector side of the base region will be swept into the collector as collector current.

$$\begin{aligned}
 i_C &= -I_n \\
 &= A_E q D_n \frac{n_p(0)}{W} \\
 &= \frac{A_E q D_n n_{p0}}{W} e^{v_{BE}/V_T} \\
 &= \frac{A_E q D_n n_i^2}{W N_A} e^{v_{BE}/V_T} \\
 &= I_S e^{v_{BE}/V_T}
 \end{aligned}$$

- Saturation current (also known as scale current) $I_S = (A_E q D_n n_i^2) / (W N_A)$
 - * A strong function of temperature.
 - * Proportional to the cross-sectional area of the base-emitter junction.
 - * Inverse proportional to the base width W .

- Base current i_B
 - i_B is composed of two currents.
 - * The holes injected from the base region into the emitter region.

$$i_{B1} = \frac{A_E q D_p n_i^2}{N_D L_p} e^{v_{BE}/V_T}$$

- * The holes that have to be supplied by the external circuit due to the recombination.
 - τ_b is the average time for a minority electron to recombine with a majority hole.

$$i_{B2} = \frac{1}{2} \frac{A_E q W n_i^2}{\tau_b N_A} e^{v_{BE}/V_T}$$

- Formulation of i_B in terms of i_C .
 - * I_S is the saturation current of i_C (refer to Eq.(7.4))
 - * $\beta = 1 / \left(\frac{D_p}{D_n} \frac{N_A}{N_D} \frac{W}{L_p} + \frac{1}{2} \frac{W^2}{D_n \tau_b} \right)$ is a constant (normally in the range $50 \sim 200$) for a given transistor.
 - * β is mainly influenced by (1) the width of the base region, and (2) the relative dopings of the base region and the emitter region $\frac{N_A}{N_D}$.
 - To achieve high β values, the base should be thin (W small) and lightly doped, and the emitter heavily doped.

$$\begin{aligned}
 i_B &= i_{B1} + i_{B2} \\
 &= I_S \left(\frac{D_p}{D_n} \frac{N_A}{N_D} \frac{W}{L_p} + \frac{1}{2} \frac{W^2}{D_n \tau_b} \right) e^{v_{BE}/V_T} \\
 &= \left(\frac{D_p}{D_n} \frac{N_A}{N_D} \frac{W}{L_p} + \frac{1}{2} \frac{W^2}{D_n \tau_b} \right) i_C \\
 &= \frac{1}{\beta} \times i_C
 \end{aligned}$$

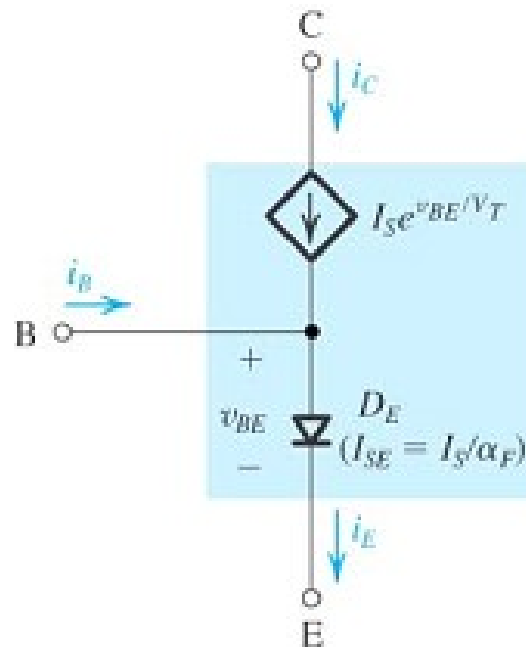
- Emitter current i_E
 - From KCL, the i_E and i_C can be related as follows:

$$\begin{aligned}i_E &= i_B + i_C \\&= \frac{1}{\beta} i_C + i_C \\&= \frac{1 + \beta}{\beta} \times i_C \\&= \frac{1}{\alpha} \times i_C \\&= \frac{1}{\alpha} \times I_s e^{v_{BE}/V_T}\end{aligned}$$

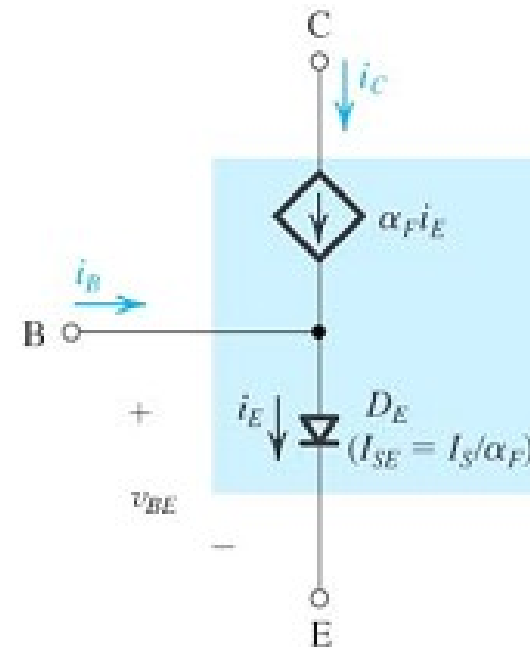
- * $\alpha = \beta / (1 + \beta) \simeq 1$ is a constant for a given transistor.
- * Small change in α corresponds to large changes in β .

- Recapitulation
 - Configuration
 - * EBJ (Forward), CBJ (Reverse)
 - Relationship between i_C , i_B , and i_E .
 - * $i_C = \beta \times i_B$.
 - β (normally in the range 50~200) is a constant for a given transistor.
 - * $i_C = \alpha \times i_E$.
 - α ($\beta / (1 + \beta) \lesssim 1$) is a constant for a given transistor.
 - * i_B , i_C , and i_E are all controlled by v_{BE} .

$$\begin{aligned}
 i_C &= I_S e^{v_{BE}/V_T} \\
 i_B &= \frac{1}{\beta} I_S e^{v_{BE}/V_T} \\
 i_E &= \frac{1}{\alpha} I_S e^{v_{BE}/V_T}
 \end{aligned}$$



(a)



(b)

Large signal equivalent model of the NPN BJT operating in the forward active mode.

- * i_C behaves as a voltage (v_{BE}) controlled current source.

$$i_C + i_B = i_E = \frac{1}{\alpha} i_C$$

- * i_C behaves as a current (i_E) controlled current source.

$$\begin{aligned} i_C + i_B &= i_E \\ \Rightarrow \alpha i_E + i_B &= i_E \end{aligned}$$

- * The diode D_E represents the forward base-emitter junction.

Discussion About BJT in the Active Mode

1. The proportion of electrons from the emitter that “make it” to the base is called the **collector efficiency, α** :

$$\alpha \equiv \frac{i_C}{i_E} \quad \text{or} \quad \boxed{i_C = \alpha i_E} \quad (1)$$

Typically it has values of near 0.99. Note that α is called the common-base current gain in the text.

$$i_E = i_B + i_C \quad (2)$$

or
$$i_B = i_E - i_C = (1 - \alpha) i_E \quad (3)$$

We can deduce from (3) that with $\alpha \approx 1$, then i_B will be much, much smaller than i_E .

2. The ratio of collector current to base current is called the **current gain, β** :

$$\beta \equiv \frac{i_C}{i_B} \quad \text{or} \quad \boxed{i_C = \beta i_B} \quad (4)$$

3. Dividing (1) by (4) we find that

$$\frac{i_C}{i_B} = \frac{\alpha}{1 - \alpha}$$

Equating this to (4) we find

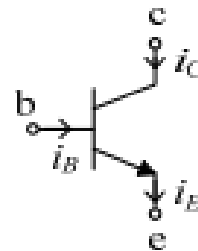
$$\boxed{\beta = \frac{\alpha}{1 - \alpha}}$$

and solving this equation for α

$$\boxed{\alpha = \frac{\beta}{\beta + 1}}$$

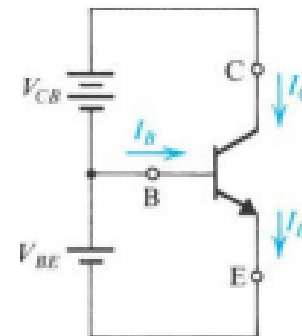
With $\alpha \approx 0.99 \Rightarrow \beta \approx 100$.

4. One can think of the **base current in the BJT as controlling the collector current**: $i_C = \beta i_B$. Since β is large, then a small change in i_B produces a large change in i_C . If the base were an input signal and the collector the output, then this would be signal amplification! Awesome!
5. The circuit symbol and current conventions for the *npn* BJT are



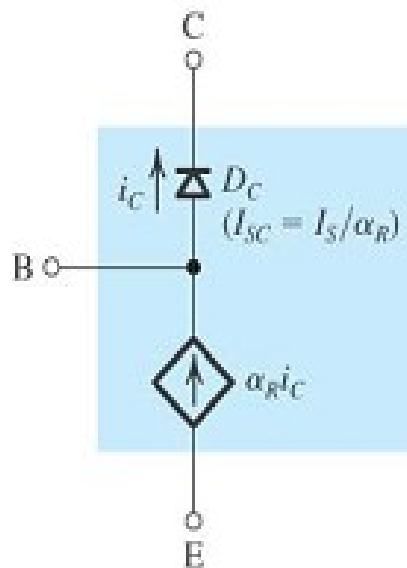
The arrows indicate the assumed directions for positive current for the *npn* BJT. The **filled arrow** is always located on the emitter and helps us to remember the direction of the emitter current.

6. one biasing circuit might be



Reverse Active Mode

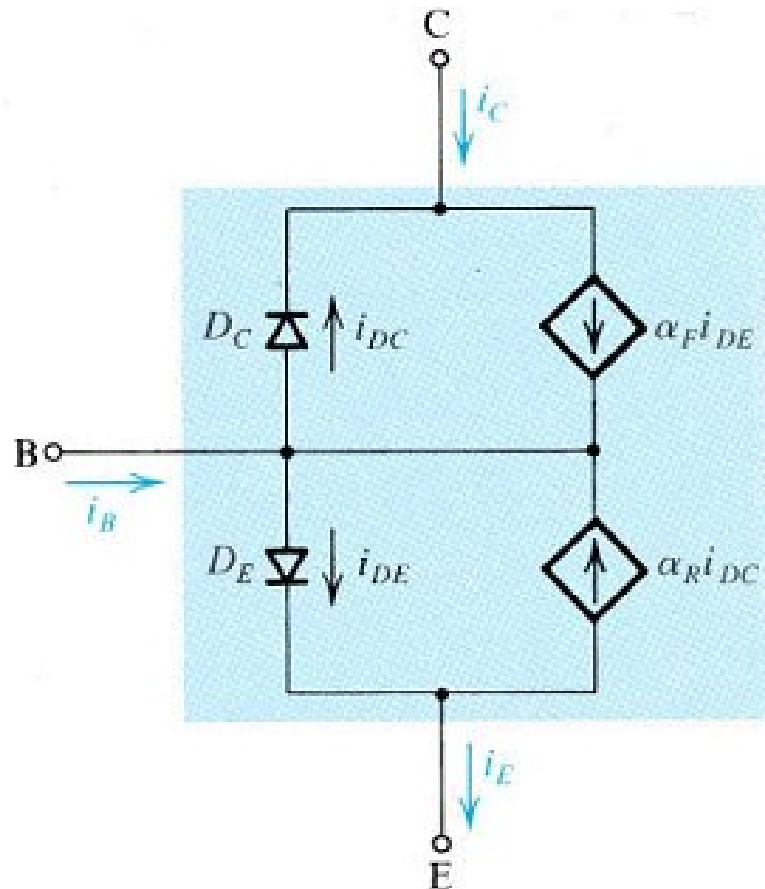
- The α and β in the reverse active mode are much lower than those in the forward active mode.
 - α_R is in the range of 0.01 to 0.5.
 - * In forward active mode, the collector virtually surrounds the emitter region.
 - Electrons injected into the thin base region are mostly captured by the collector.
 - * In reverse active mode, the emitter virtually surrounds the collector region.
 - Electrons injected into the thin base region are partly captured by the collector.
 - β_R is in the range of 0.01 to 1.
- CBJ has a much larger area than EBJ.
 - The diode D_C denotes the forward base-collector junction.
 - The diode D_C has larger scale current (I_{SC}) than D_E does.
 - * The diode D_C has lower voltage drop when forward biased.



Large signal equivalent model of the NPN BJT operating in the reverse active mode.

Ebers-Moll (EM) Model

- A composite model that can be used to predict the operations of the BJT in all possible modes.
- α and β
 - α_F and β_F denotes the parameters in forward active mode.
 - α_R and β_R denotes the parameters in reverse active mode.



Ebers-Moll model of the NPN transistor.

- Equivalent saturation current I_{SE} and I_{SC}
 - I_{SE} and I_{SC} are the equivalent saturation currents at the EBJ and CBJ, respectively.

$$I_{SE} = \frac{1}{\alpha_F} I_S$$

$$I_{SC} = \frac{1}{\alpha_R} I_S$$

$$\Rightarrow \alpha_F I_{SE} = \alpha_R I_{SC} = I_S$$

- i_C , i_B , and i_E in the EM model

$$i_E = i_{DE} - \alpha_R i_{DC}$$

$$i_C = -i_{DC} + \alpha_F i_{DE}$$

$$i_B = (1 - \alpha_F) i_{DE} + (1 - \alpha_R) i_{DC}$$

$$- i_{DE} = I_{SE} (e^{v_{BE}/V_T} - 1).$$

$$- i_{DC} = I_{SC} (e^{v_{BC}/V_T} - 1).$$

$$i_E = \frac{I_S}{\alpha_F} (e^{v_{BE}/V_T} - 1) - I_S (e^{v_{BC}/V_T} - 1)$$

$$i_C = I_S (e^{v_{BE}/V_T} - 1) - \frac{I_S}{\alpha_R} (e^{v_{BC}/V_T} - 1)$$

$$i_B = \frac{I_S}{\beta_F} (e^{v_{BE}/V_T} - 1) + \frac{I_S}{\beta_R} (e^{v_{BC}/V_T} - 1)$$

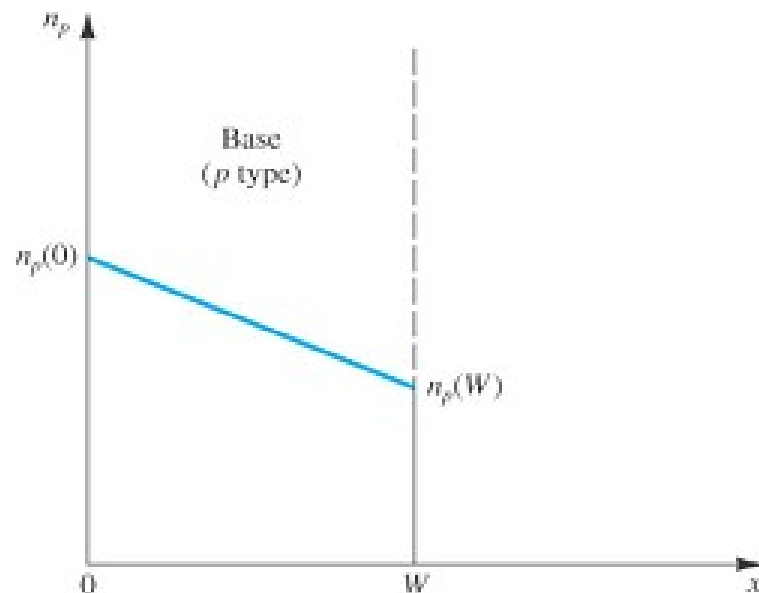
$$- \beta_F = \alpha_F / (1 - \alpha_F).$$

$$- \beta_R = \alpha_R / (1 - \alpha_R).$$

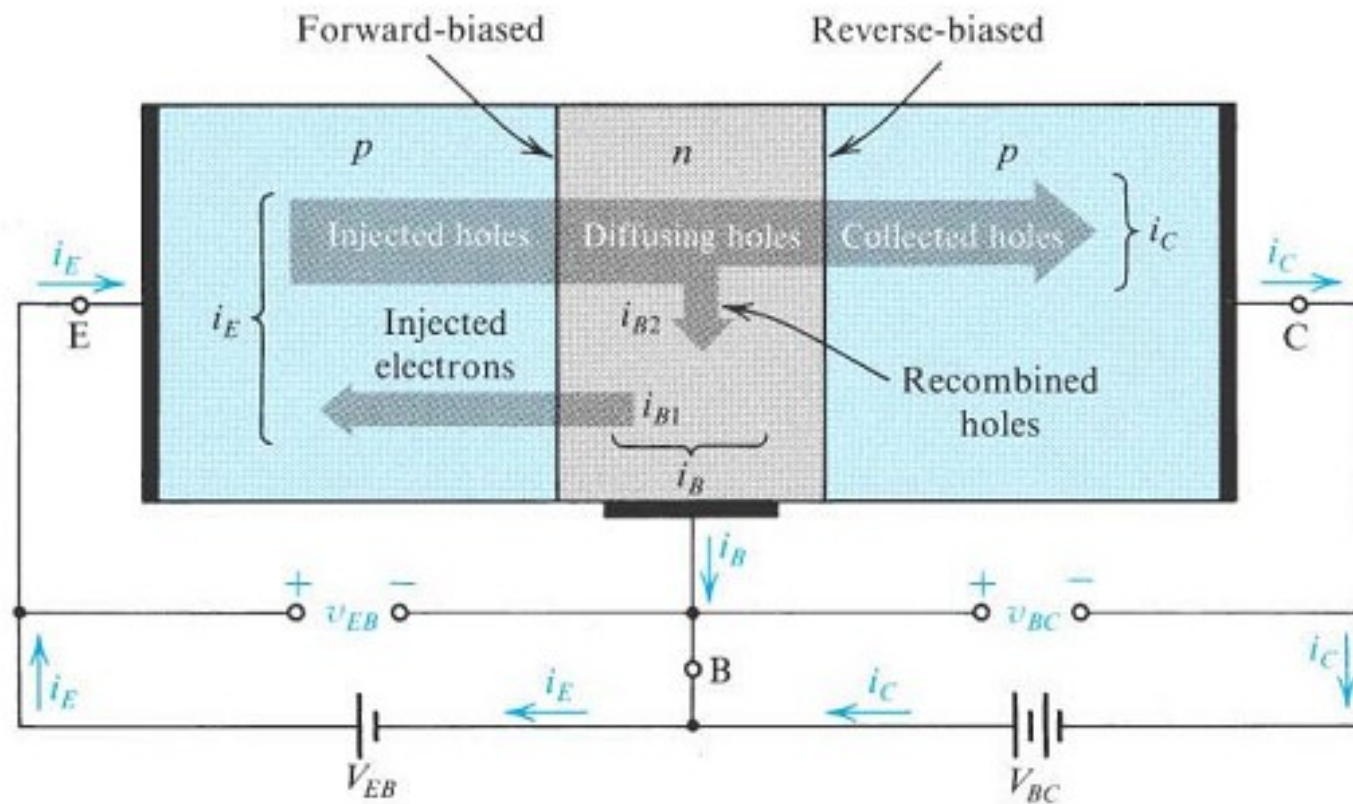
Saturation Mode

- CBJ is in forward bias, i.e., $v_{BC} > 0.4V$.
 - CBJ has larger junction area than EBJ.
 - * CBJ has larger saturation current I_S and lower cut-in voltage than EBJ.
 - * In forward bias,
 - The voltage drop across CBJ is $0.4V$.
 - The voltage drop across EBJ is $0.7V$.
 - As v_{BC} is increased, i_C will be decreased and eventually reach zero.

$$i_C \simeq I_S e^{v_{BE}/V_T} - \frac{I_S}{\alpha_R} e^{v_{BC}/V_T}$$



Concentration profile of the minority carriers in the base region of an NPN transistor.



Current flow in a PNP transistor biased to operate in the active mode.

Operations of PNP Transistor

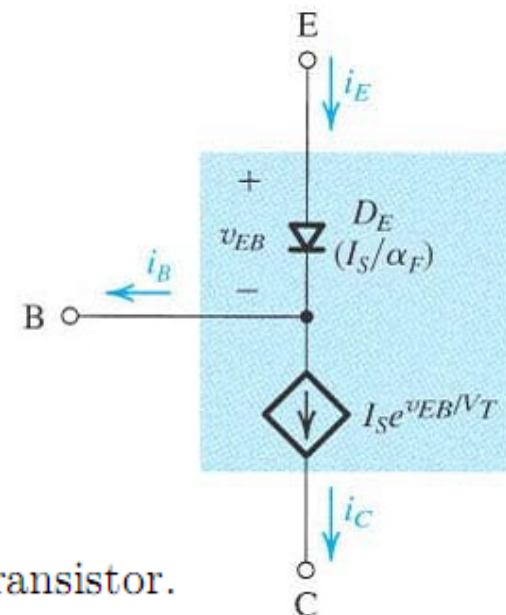
Active Mode

- Current in a PNP transistor is mainly conducted by holes.
- Emitter-Base Junction
 - Forward bias, $v_{EB} > 0$.
 - Holes in the emitter region are injected into the base causing a current i_{E1} .
 - Electrons in the base region are injected into the emitter region causing a current i_{E2} .
 - * Generally, $i_{E1} \gg i_{E2}$.

$$i_E(t) = i_{E1} + i_{E2}$$

- Base region
 - Tapered concentration causes the holes to diffuse through the base region toward the collector.
 - * Some of the holes may combine with the electrons.
 - * The recombination process is quite small due to lightly doped and thin base region.

- Collector-Base Junction
 - Reverse bias, $v_{BC} > 0$.
 - The holes near the collector side are swept into the collector region causing zero concentration at the collector side.
- Collector current, i_C .
 - Most of the diffusing holes will reach collector region.
 - * Only a very small percentage of holes are recombined with the electrons in the base region.
 - As long as $v_{BC} > 0$, i_C is independent of v_{BC} .
 - * The holes that reach the collector side of the base region will be swept into the collector as collector current.

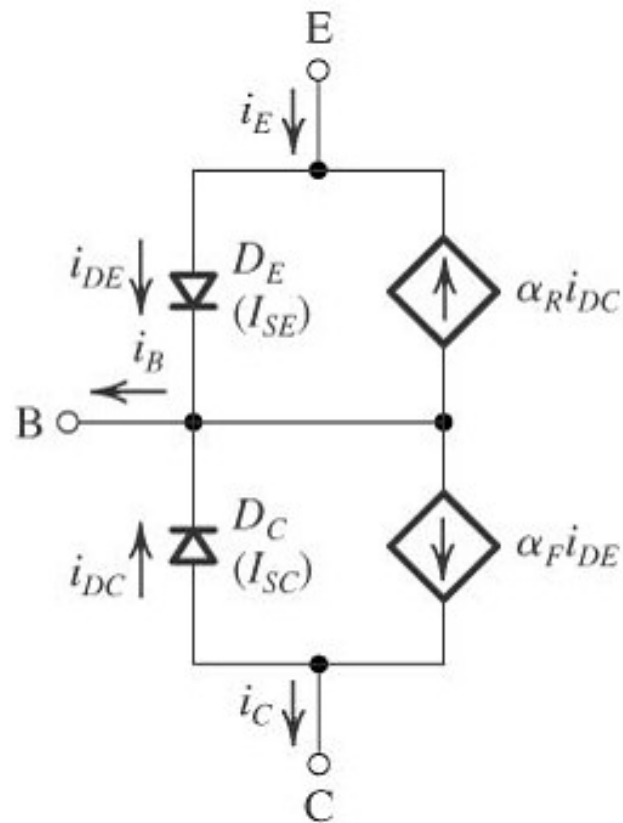


the large signal equivalent model of the PNP transistor.

- Base current i_B
 - i_B is composed of two currents.
 - * The electrons injected from the base region into the emitter region.
 - * The electrons that have to be supplied by the external circuit due to the recombination.
- Emitter current i_E
 - From KCL, the i_E and i_C can be related as follows:

$$\begin{aligned}
 i_E &= i_B + i_C \\
 &= \frac{1}{\beta} i_C + i_C \\
 &= \frac{1 + \beta}{\beta} \times i_C \\
 &= \frac{1}{\alpha} \times i_C \\
 &= \frac{1}{\alpha} \times I_s e^{v_{EB}/V_T}
 \end{aligned}$$

- * $\alpha = \beta / (1 + \beta) \simeq 1$ is a constant for a given transistor.
- * Small change in α corresponds to large changes in β .



Ebers-Moll model of the PNP transistor.

Reverse Active Mode

- Similar to NPN transistor.

Saturation Mode

- Similar to NPN transistor.

Summary of the i_C , i_B , i_E Relationships in Active Mode

- NPN transistor

$$i_c = I_s e^{v_{BE}/V_T}$$

$$i_B = \frac{I_s}{\beta} e^{v_{BE}/V_T}$$

$$i_E = \frac{I_s}{\alpha} e^{v_{BE}/V_T}$$

$$i_C = \alpha i_E$$

$$i_C = \beta i_B$$

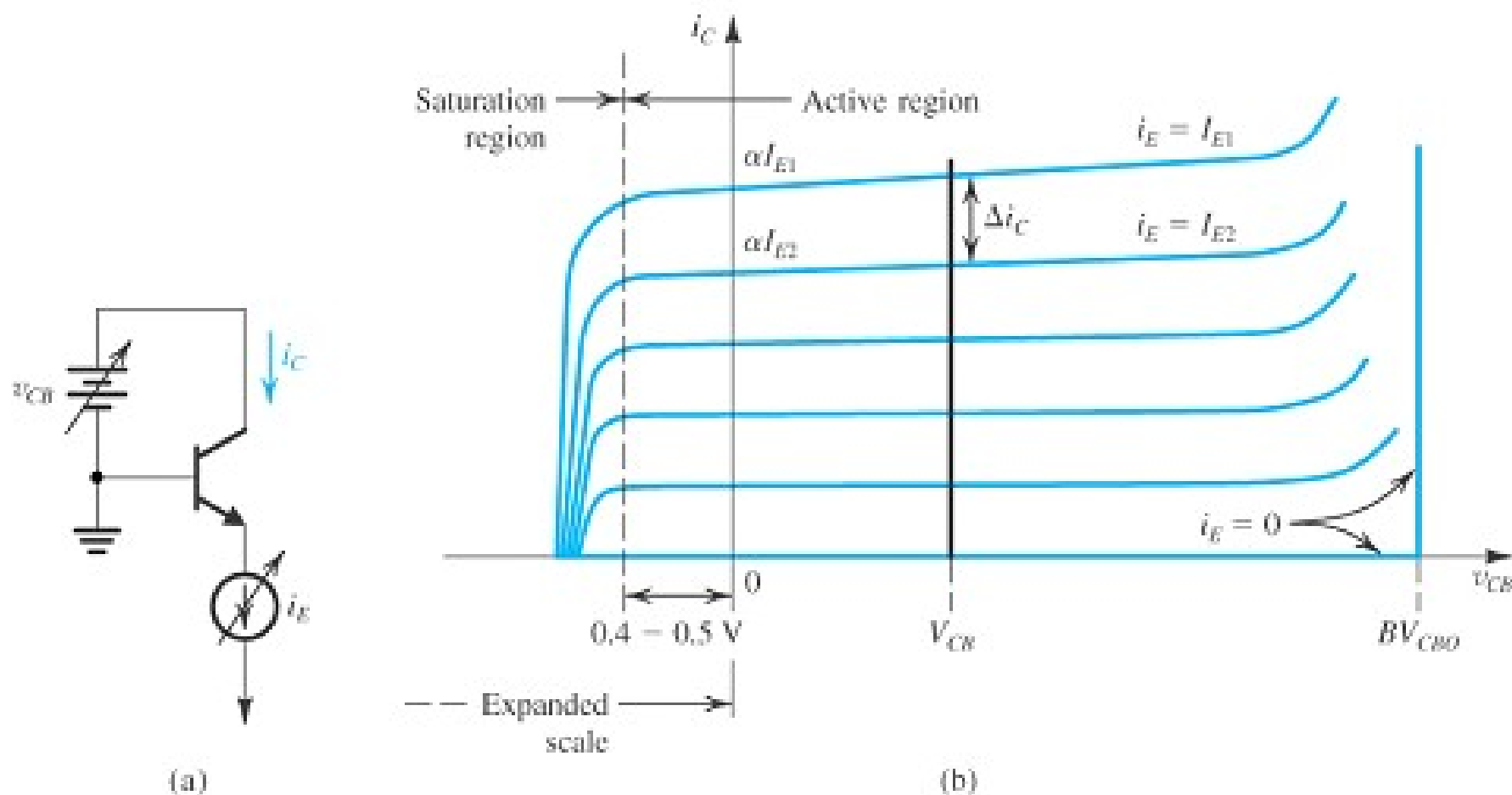
$$i_B = (1 - \alpha) i_E = \frac{i_E}{1 + \beta}$$

$$i_E = (1 + \beta) i_B$$

- PNP transistor.
 - The v_{BE} is replaced by v_{EB} .

The $i - v$ Characteristics of NPN Transistor

Common Base ($i_C - v_{CB}$)



The $i_C - v_{CB}$ characteristics of an NPN transistor.

The $i - v$ Characteristics of NPN Transistor

Common Base ($i_C - v_{CB}$)

- Figure depicts the i_C versus v_{CB} for various i_E , which is also known as the common-base characteristics.
 - Input port: emitter and base terminals.
 - * Input current i_E .
 - Output port: collector and base terminals.
 - * Output current i_C .
 - The base terminal serves as a common terminal to both input port and output port.

The $i - v$ Characteristics of NPN Transistor

Common Base ($i_C - v_{CB}$)

- Active Region ($v_{CB} \geq -0.4V$)
 - i_C depends slightly on v_{CB} and shows a small positive slope.
 - i_C shows a rapid increase, known as breakdown phenomenon, for a relatively large value of v_{CB} .
 - Each $i_C - v_{CB}$ curve intersects the vertical axis at a current level equal to αI_E .
 - * Total or large-signal α (common-base current gain)
 - $\alpha = i_C/i_E$, where i_C and i_E denote the total collector and emitter currents, respectively.
 - * Incremental or small-signal α
 - $\alpha = \Delta i_C / \Delta i_E$.
 - * Usually, the values of incremental and total α differs slightly.

The $i - v$ Characteristics of NPN Transistor

Common Base ($i_C - v_{CB}$)

- Saturation Region ($v_{CB} < -0.4V$)
 - CBJ is forward biased.
 - The EM model can be used to determine the v_{CB} at which i_C is zero.

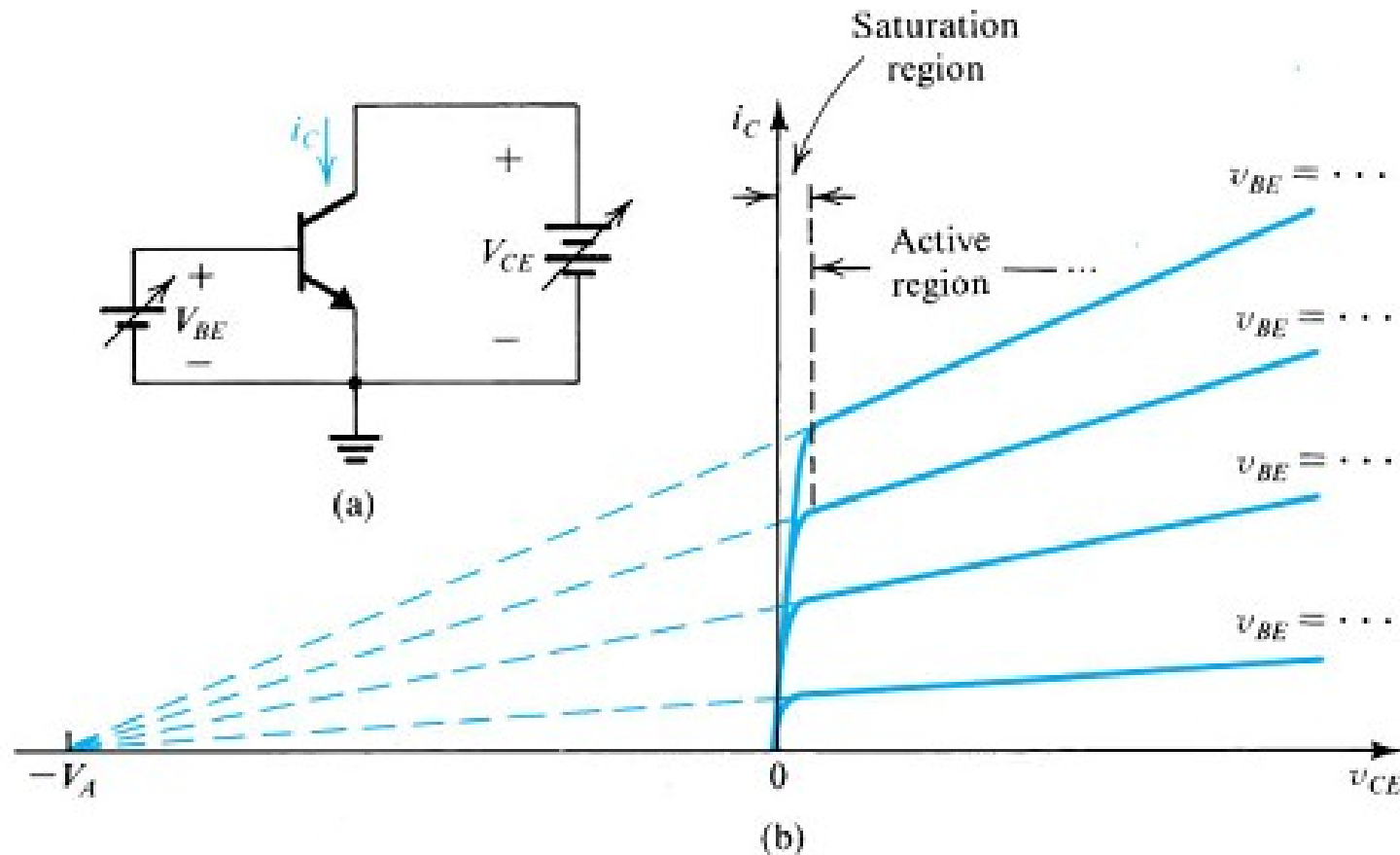
The $i - v$ Characteristics of NPN Transistor

Common Emitter ($i_C - v_{CE}$)

- Figure depicts the i_C versus v_{CE} for various v_{BE} , which is also known as the common-emitter characteristics.
 - Input port: base and emitter terminals.
 - * Input current i_B .
 - Output port: collector and emitter terminals.
 - * Output current i_C .
 - The emitter terminal serves as a common terminal to both input port and output port.

The $i - v$ Characteristics of NPN Transistor

Common Emitter ($i_C - v_{CE}$)



The $i_C - v_{CE}$ characteristics of the BJT.

The $i - v$ Characteristics of NPN Transistor

Common Emitter ($i_C - v_{CE}$)

- Active Region ($v_{CB} \geq -0.4V$)
 - i_C increases as the v_{CE} is increased, which is known as Early Effect.
 - * At a given v_{BE} , increasing v_{CE} increases the width of the depletion region of the CBJ.
 - * The effective base width W is decreased.
 - * I_S is inversely proportional to the base width W .
 - When extrapolated, the characteristics line meet at point on the negative v_{CE} (normally in the range of 50V to 100V), $-V_A$.
 - * V_A is a constant for a given transistor.

The $i - v$ Characteristics of NPN Transistor

Common Emitter ($i_C - v_{CE}$)

- Large signal equivalent circuit model in active mode.
 - The linear dependency of i_C on v_{CE} can be formulized as follows:

$$i_C = I_S e^{v_{BE}/V_T} \left(1 + \frac{v_{CE}}{V_A}\right) = I_C \left(1 + \frac{v_{CE}}{V_A}\right)$$

- The output resistance looking into the collector-emitter terminals.
 - * Inversely proportional to the collector current I_C without considering Early effect.
 - * Controlled by v_{BE} .

$$\begin{aligned} \Delta i_C &= I_S e^{v_{BE}/V_T} \left(\frac{\Delta v_{CE}}{V_A} \right) \\ \Rightarrow r_o &= \frac{\Delta v_{CE}}{\Delta i_C} = \frac{V_A}{I_C} \end{aligned}$$

The $i - v$ Characteristics of NPN Transistor

Common Emitter ($i_C - v_{CE}$)

- Figure 7.18 depicts the large signal equivalent circuit model of an NPN BJT in the active mode and with the common emitter configuration.
 - * Figure 7.18 (a), voltage v_{BE} controls the collector current source.
 - * Figure 7.18 (b), the base current i_B controls the collector current source $\beta \times i_B$.

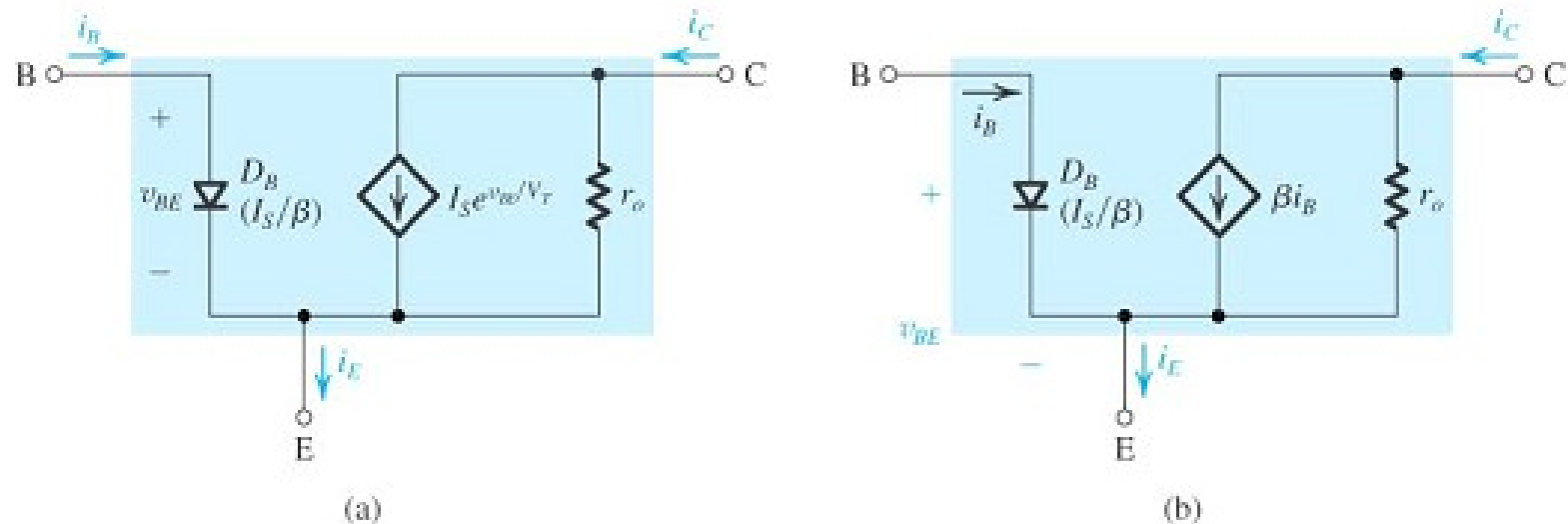


Figure 7.18: Large signal equivalent circuit model of an NPN BJT operating in the active mode and with common-emitter configuration.

The $i - v$ Characteristics of NPN Transistor

Common Emitter ($i_C - v_{CE}$)

- Large signal or DC β
 - * The ratio of total current in the collector to the total current in the base, which represents the ideal current gain (where r_o is not present) of the common-emitter configuration.

$$\beta_{dc} = \frac{i_C}{i_B} \Big|_{v_{CE}=\text{constant}}$$

- * β is also known as the common-emitter current gain.
- Incremental or AC β
 - * Short-circuit common-emitter current gain.
 - * AC β and DC β differ approximately 10% to 20%.

$$\beta_{ac} = \frac{\Delta i_C}{\Delta i_B} \Big|_{v_{CE}=\text{constant}}$$

The $i - v$ Characteristics of NPN Transistor

Common Emitter ($i_C - v_{CE}$)

- Saturation Region ($v_{CB} < -0.4V$)
 - Figure depicts an expanded view of the common-emitter characteristic in the saturation region.
 - Analytical expressions of $i_C - v_{CE}$ using EM model.
 - * $v_{BE} = v_{CE} + v_{CB}$.

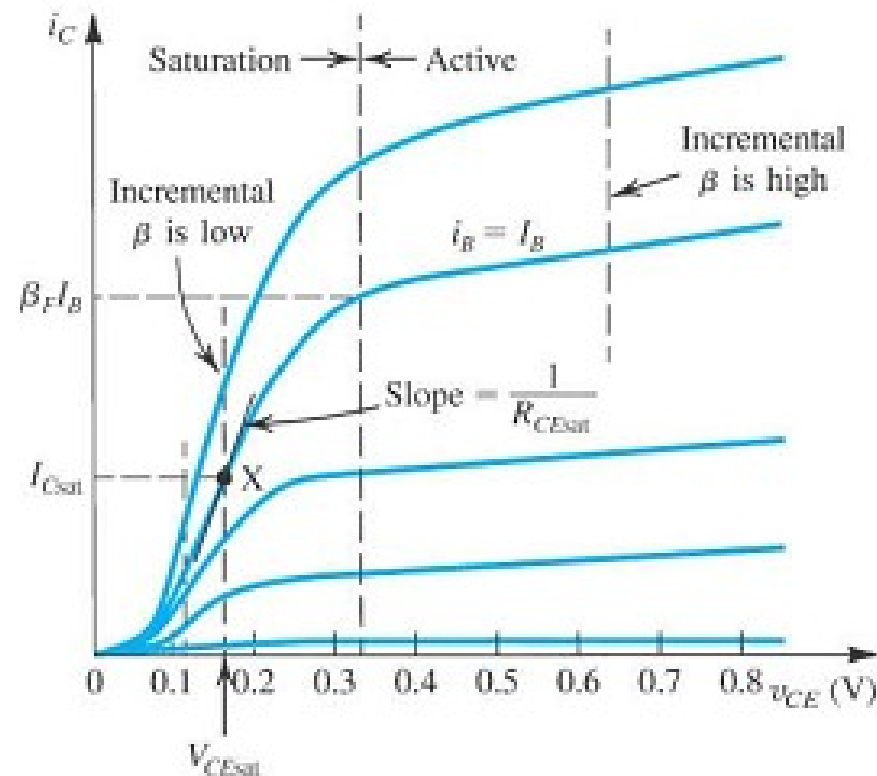
$$i_C \simeq I_S(e^{v_{BE}/V_T}) - \frac{I_S}{\alpha_R}(e^{v_{BC}/V_T})$$

$$I_B \simeq \frac{I_S}{\beta_F}(e^{v_{BE}/V_T}) + \frac{I_S}{\beta_R}(e^{v_{BC}/V_T})$$

$$i_C \simeq (\beta_F I_B) \left(\frac{e^{v_{CE}/V_T} - \frac{1}{\alpha_R}}{e^{v_{CE}/V_T} - \frac{\beta_F}{\beta_R}} \right)$$

The $i - v$ Characteristics of NPN Transistor

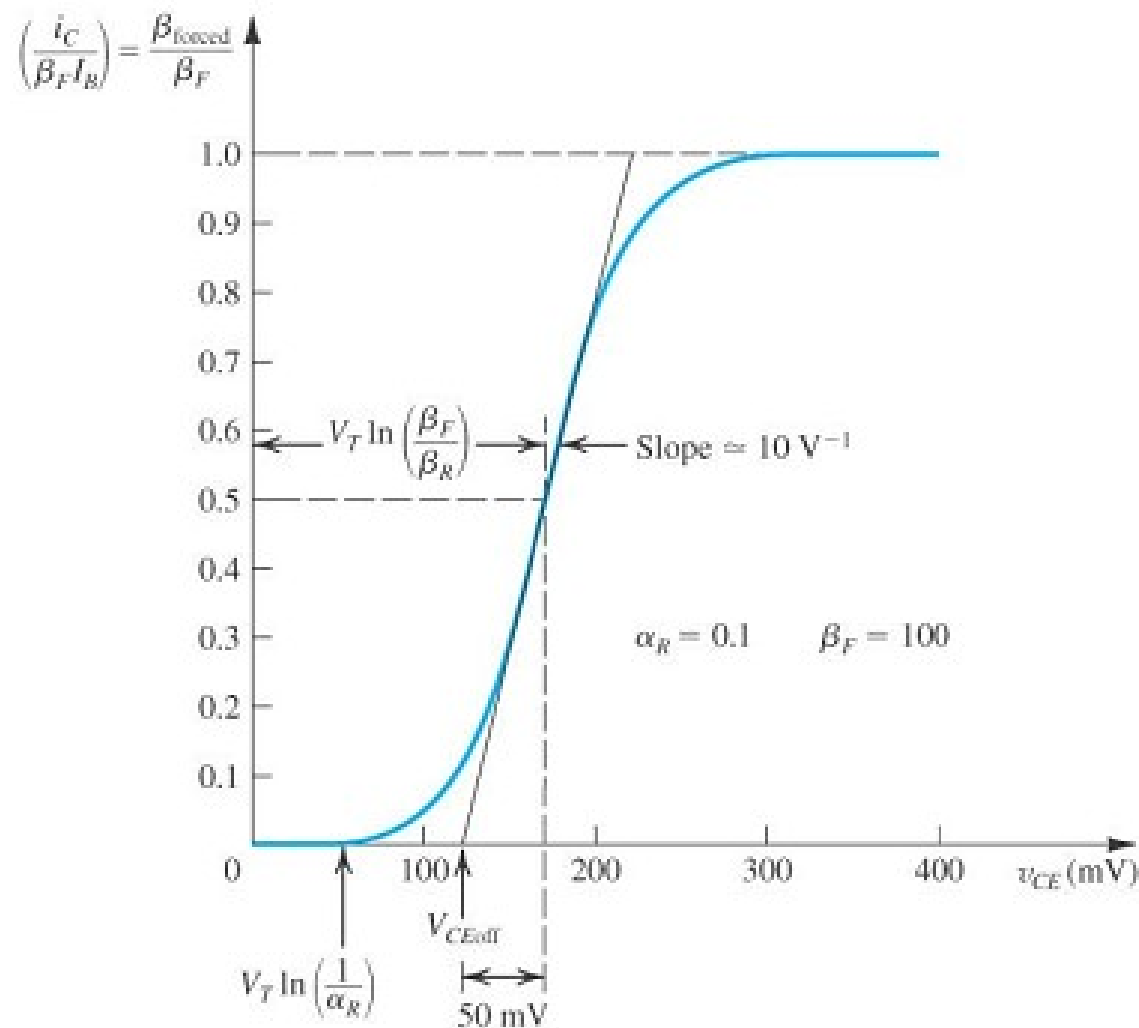
Common Emitter ($i_C - v_{CE}$)



An expanded view of the common-emitter characteristic in the saturation region.

The $i - v$ Characteristics of NPN Transistor

Common Emitter ($i_C - v_{CE}$)



Plot of normalized i_C versus v_{CE} for an NPN transistor with $\beta_F = 100$ and $\alpha_R = 0.1$.

The $i - v$ Characteristics of NPN Transistor

Common Emitter ($i_C - v_{CE}$)

- Large signal equivalent circuit model in saturation mode.
 - The saturation transistor exhibits a low collector-to-emitter resistance R_{CEsat} .

$$R_{CEsat} = \left. \frac{\partial v_{CE}}{\partial i_C} \right|_{i_B=I_B, i_C=I_C} \simeq 1/10\beta_F I_B$$

- At the collector side, the transistor is modeled as a resistance R_{CEsat} in series with a battery v_{CEoff} as shown in Figure 7.21 (c).
 - * V_{CEoff} is typically around $0.1V$.
 - * V_{CEsat} is typically around $0.1 \sim 0.3V$.

$$V_{CEsat} = V_{CEoff} + I_{Csat}R_{CEsat}$$

- For many applications, the even simpler model shown in Figure 7.21 is used.

The $i - v$ Characteristics of NPN Transistor

Common Emitter ($i_C - v_{CE}$)

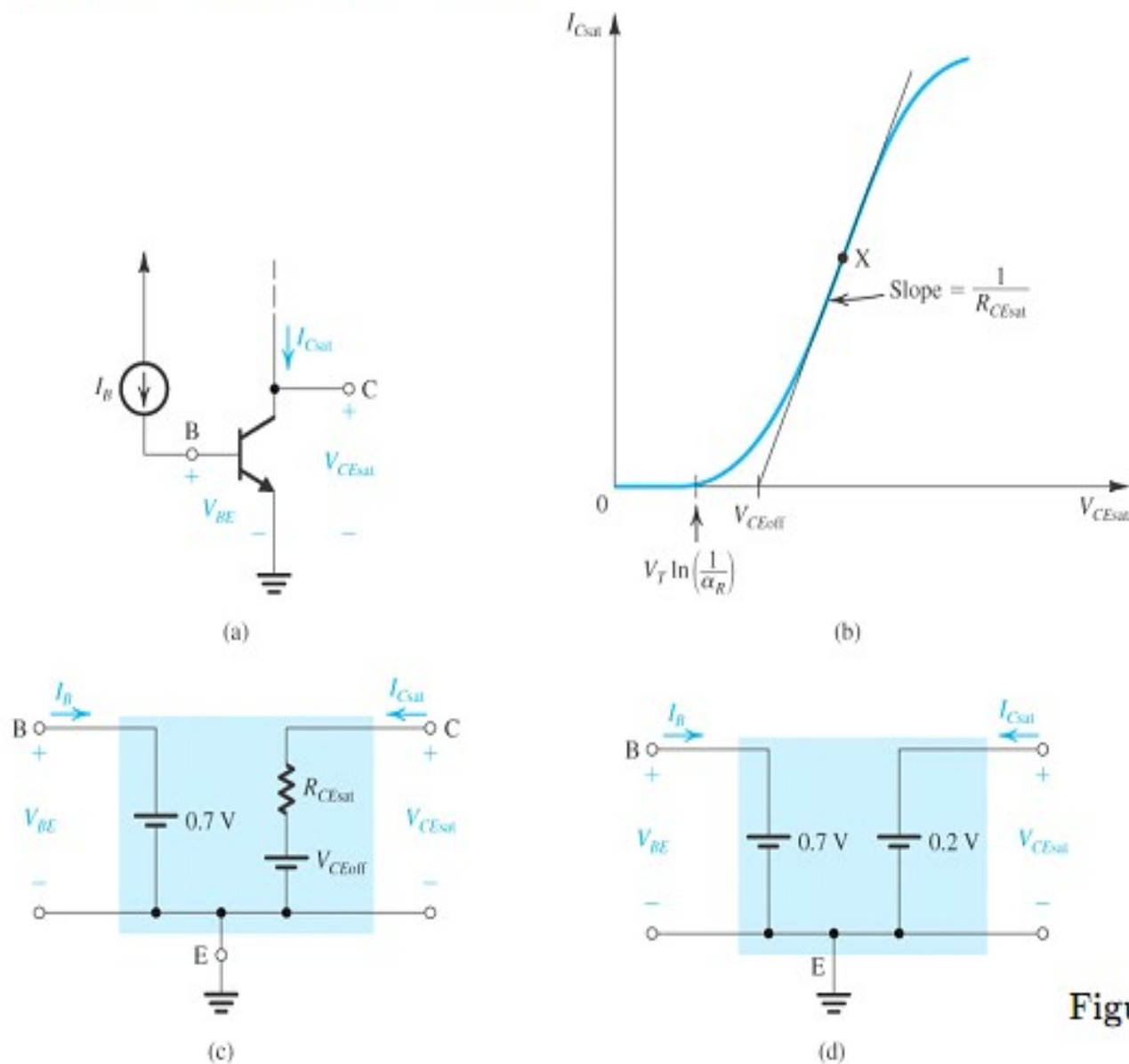


Figure 7.21: Equivalent circuit representation of the saturated transistor.