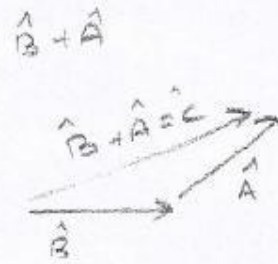
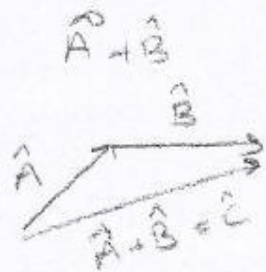


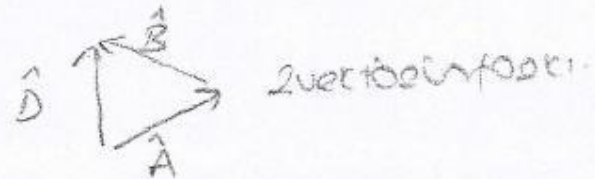
i) Vektörlerin toplaması



Birleşme özelliği

$$(\hat{A} + \hat{B}) + \hat{C} = \hat{A} + (\hat{B} + \hat{C})$$

$$\hat{A} - \hat{B} = \hat{A} + (-\hat{B})$$



ii) Bir skalar ile çarpma

$$\alpha \cdot \hat{A} \Rightarrow \xrightarrow{2}$$

$$\alpha \cdot \hat{B} \Rightarrow \xrightarrow{6}$$

$$\alpha \cdot (\hat{A} + \hat{B}) = \alpha \hat{A} + \alpha \hat{B} \quad \alpha \text{ negatif ise vektörün yönü ters olur}$$

iii) İki vektörün çarpımında nokta çarpımı (dot çarpım)

$$\hat{A} \cdot \hat{B} = A \cdot B \cdot \cos \theta \rightarrow \text{vektör arasındaki açıdır.}$$

$$\hat{A} \cdot \hat{B} = \hat{B} \cdot \hat{A}$$

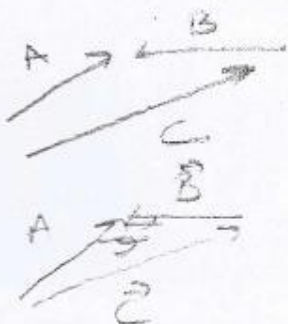
$$\hat{A} \cdot (\hat{B} + \hat{C}) = \hat{A} \cdot \hat{B} + \hat{A} \cdot \hat{C}$$

$$\hat{A} \cdot \hat{A} = A \cdot A \cdot \cos 0 = A^2$$



$$A \cdot B = A \cdot B \cdot \cos 90^\circ = 0 //$$

ÖRNEK $\hat{C} = \hat{A} - \hat{B}$, $\hat{C}$ 'nin kendisi ile noktasal çarpımını $\hat{A}$ ve $\hat{B}$ cinsinden hesaplayınız



$$\hat{C} \cdot \hat{C} = C^2 = (\hat{A} - \hat{B}) \cdot (\hat{A} - \hat{B})$$

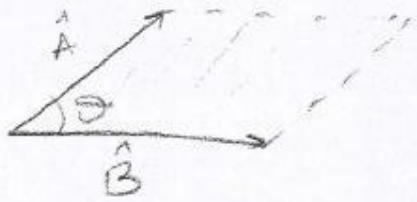
$$\hat{A} \cdot \hat{A} - \hat{A} \cdot \hat{B} - \hat{B} \cdot \hat{A} + \hat{B} \cdot \hat{B}$$

$$A^2 + B^2 - 2AB$$

$$A^2 + B^2 - 2AB \cos \theta$$

$$C^2 = A^2 + B^2 - 2AB \cos \theta$$

iv) 2 vektörün cos cospmı (vektörel cospm)



$\hat{A} \times \hat{B}$ olarak vektörün
yönü sayfa dış binih line
doğru duz.

$\rightarrow \otimes \quad \odot \rightarrow$ dışa.

$$C = A \cdot B \cdot \sin \theta$$

$\hat{A}$ ve $\hat{B}$ vektörlerinden duzon paralel kere n olan $\hat{C}$ nin
bysklugne esit

$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

$\hat{n}$ = biyim vektör

$$\hat{A} \times \hat{B} = A \cdot B \cdot \sin \theta \cdot \hat{n} \quad |\hat{n}| = 1 \quad \text{bysklugu} = 1.$$

$\hat{n}$ yönü goğ elluwall ne belikinin Butonere saypmn.
rine doğru duz. A ve B nin duzluallıgu duzleme dırtır.

$$\hat{A} \times (\hat{B} + \hat{C}) = \hat{A} \times \hat{B} + \hat{A} \times \hat{C}$$

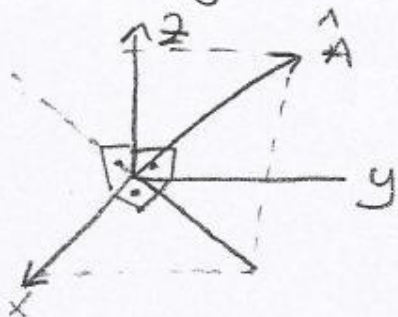
$$\hat{A} \times \hat{A} = 0 \quad (\sin 0 = 0)$$

$$\begin{array}{c} \xrightarrow{\hat{A}} \\ \xrightarrow{\hat{B}} \end{array} \quad \hat{A} \times \hat{B} = 0.$$

Vektörlerin biyim vektörler dınsından ifade
edilmesi

vektör cebiri: Biregenlerin kullonımı

kartezyen koordinat sistemı



$$\hat{A} = A_x \cdot \hat{x} + A_y \cdot \hat{y} + A_z \cdot \hat{z}$$

$\hat{x}, \hat{y}, \hat{z}$ biyim vektörler

$$|\hat{x}| = |\hat{y}| = |\hat{z}| = 1.$$

A_x, A_y, A_z skaler değerkler.

A_x $\hat{A}$ nin x koordinat etserine ısdırsımın bysklugu

4 vektör izleminin birim vektöre uygulanması

1) Toplama

$$\hat{A} + \hat{B} = (Ax + Bx)\hat{x} + (Ay + By)\hat{y} + (Az + Bz)\hat{z}$$

$$\hat{A} = Ax \cdot \hat{x} + Ay \cdot \hat{y} + Az \cdot \hat{z}$$

$$\hat{B} = Bx \cdot \hat{x} + By \cdot \hat{y} + Bz \cdot \hat{z}$$

2) skaler çarpım

$$\alpha \cdot \hat{A} = \alpha Ax \cdot \hat{x} + \alpha Ay \cdot \hat{y} + \alpha Az \cdot \hat{z}$$

3) vektörel çarpım

$$\hat{A} \cdot \hat{B} = (Ax \cdot \hat{x} + Ay \cdot \hat{y} + Az \cdot \hat{z}) \cdot (Bx \cdot \hat{x} + By \cdot \hat{y} + Bz \cdot \hat{z})$$

$$= Ax \cdot Bx \cdot \hat{x} \cdot \hat{x} + Ax \cdot By \cdot \hat{x} \cdot \hat{y} + Ax \cdot Bz \cdot \hat{x} \cdot \hat{z} + Ay \cdot Bx \cdot \hat{y} \cdot \hat{x} + Ay \cdot By \cdot \hat{y} \cdot \hat{y} + Ay \cdot Bz \cdot \hat{y} \cdot \hat{z} + Az \cdot Bx \cdot \hat{z} \cdot \hat{x} + Az \cdot By \cdot \hat{z} \cdot \hat{y} + Az \cdot Bz \cdot \hat{z} \cdot \hat{z}$$

$$|\hat{x}| = 1$$

$$\hat{x} \cdot \hat{x} = x^2 = 1$$

$$\hat{y} \cdot \hat{y} = y^2 = 1$$

$$\hat{z} \cdot \hat{z} = z^2 = 1$$

$$\hat{x} \cdot \hat{y} = 0$$

$$\hat{y} \cdot \hat{z} = 0$$

$$\hat{z} \cdot \hat{x} = 0$$

$$\boxed{Ax \cdot Bx + Ay \cdot By + Az \cdot Bz}$$

$$\hat{A} \cdot \hat{A} = Ax \cdot Ax + Ay \cdot Ay + Az \cdot Az$$

$$A^2 = Ax^2 + Ay^2 + Az^2$$

$$A = \sqrt{Ax^2 + Ay^2 + Az^2} \Rightarrow \text{pisagor teoremi}$$

cos çarpım

$$\begin{aligned} \hat{A} \times \hat{B} &= (Ax \cdot \hat{x}) \times (Bx \cdot \hat{x}) + (Ax \cdot \hat{x} + By \cdot \hat{y}) \times (Ax \cdot \hat{x} + Bz \cdot \hat{z}) \\ &+ (Ay \cdot \hat{y}) \times (Bx \cdot \hat{x}) + (Ay \cdot \hat{y} \times By \cdot \hat{y}) + (Ay \cdot \hat{y} + Bz \cdot \hat{z}) \times \\ &+ (Az \cdot \hat{z}) \times (Bx \cdot \hat{x}) + (Az \cdot \hat{z} \times By \cdot \hat{y}) + (Az \cdot \hat{z} + Bz \cdot \hat{z}) \end{aligned}$$

$$Ax \cdot \hat{x} \times Bx \cdot \hat{x} = Ax \cdot Bx \cdot \sin \theta$$

$$\hat{x} \cdot \hat{x} = 0$$

$$\hat{x} \times \hat{y} = 1 \cdot 1 \cdot \sin 90^\circ = \hat{z}$$

$$\hat{x} \times \hat{z} = -\hat{y}$$

$$\hat{y} \times \hat{y} = 0$$

$$\hat{z} \times \hat{x} = \hat{y}$$

$$\hat{z} \times \hat{z} = 0$$

$$\hat{A} \times \hat{B} = A_x \cdot B_y \cdot \hat{z} + A_x \cdot B_z (-y) + A_y \cdot B_x (-z) + A_y \cdot B_z \cdot \hat{x} +$$

$$A_z \cdot B_x y + A_z \cdot B_y (-x)$$

$$= (A_y \cdot B_z - A_z \cdot B_y) \cdot \hat{x} + (A_z \cdot B_x - A_x \cdot B_z) \cdot \hat{y} + (A_x \cdot B_y - A_y \cdot B_x) \cdot \hat{z}$$

$$\hat{A} \times \hat{B}$$

$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	1. satır 2 birim vektör
	2. satır 1 vektörün bileşenleri
	3. satır 2 vektörün bileşenleri

$$\hat{x}(A_y B_z - B_y A_z) - \hat{y}(A_x B_z - A_z B_x) + \hat{z}(A_x B_y - A_y B_x)$$

Örnek $\hat{A} \times \hat{B} \times \hat{C}$ → vektörün vektör

$\hat{A} \cdot \hat{B} \times \hat{C}$ → skalar.

Vektör vektör vektörün noktasal skalar.
İşlem önceliği cross product.

$$A \cdot \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$A = (A_x \cdot \hat{x} + A_y \cdot \hat{y} + A_z \cdot \hat{z})$$

g) $\hat{A}$ ile $\hat{B}$ arasındaki açı

$$\hat{A} \cdot \hat{B} = A \cdot B \cos \theta$$

örnek $A = \hat{x} + \hat{y}$
 $B = \hat{x} + 3\hat{y} - 2\hat{z}$
 $C = \hat{y} + 2\hat{z}$

$$|\hat{A}| = \text{boyu} = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$$

$$B = \sqrt{2} \cdot 3 \cdot \cos \theta$$

$$\cos \theta = \frac{1}{\sqrt{2}} \quad \theta = \arccos \frac{1}{\sqrt{2}} = 45^\circ$$

a) $\hat{A} + \hat{B} = 2\hat{x} + 3\hat{y} - 2\hat{z}$

b) $\hat{B} - \hat{C} = \hat{x} + \hat{y} - 4\hat{z}$

c) $4 \cdot \hat{C} = 4\hat{y} + 8\hat{z}$

n) $\hat{A} \times \hat{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 1 & 0 \\ 1 & 3 & -2 \end{vmatrix} = \hat{A} \times \hat{B} =$

d) $|B| = \sqrt{1^2 + 3^2 + (-2)^2} = \sqrt{14} = 3.74$

e) $\hat{B}$ birim vektör. $\hat{B} = \frac{B}{|B|} = \frac{\hat{x} + 3\hat{y} - 2\hat{z}}{\sqrt{14}} = \frac{1}{\sqrt{14}}\hat{x} + \frac{3}{\sqrt{14}}\hat{y} - \frac{2}{\sqrt{14}}\hat{z}$

f) $\hat{A} \cdot \hat{B} = 1 \cdot 1 + 1 \cdot 3 + 0 \cdot (-2) = 4$

Ödev $\hat{A} = \hat{x} + \hat{y}$ devamı
 $\hat{B} = \hat{x} + 2\hat{y} - 2\hat{z}$
 $\hat{C} = \hat{y} + 2\hat{z}$

i) $\hat{A} \times \hat{B}$ nin büyüklüğü

$$\hat{A} \times \hat{B} = -2\hat{x} + 2\hat{y} + \hat{z}$$

$$|\hat{A} \times \hat{B}| = \sqrt{(-2)^2 + 2^2 + 1^2} = 3$$

$$\frac{\hat{A} \times \hat{B}}{|\hat{A} \times \hat{B}|} = \frac{-2}{3}\hat{x} + \frac{2}{3}\hat{y} + \frac{1}{3}\hat{z}$$

$$\sqrt{\frac{4}{9} + \frac{4}{9} + \frac{1}{9}} = \sqrt{\frac{9}{9}} = 1$$

j) $(\hat{A} \times \hat{B}) \times \hat{C}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -2 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3\hat{x} + 4\hat{y} - 2\hat{z}$$

k) $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & -2 \\ 0 & 1 & 2 \end{vmatrix} = 1 \cdot 6 + 1 \cdot (-2) + 0 \cdot (1) = 4$

Ödev laftaya salı 4 noz

> $\hat{A} = 3\hat{x} + 2\hat{y} + \hat{z}$ $\hat{B} = \hat{x} + \hat{y} - \hat{z}$ $\hat{C} = \hat{x} + 2\hat{y} + 3\hat{z}$

a) $|\hat{A} + \hat{B} - 4\hat{C}|$ büyüklüğü.

b) $\hat{A} + 2\hat{B} - \hat{C}$ vektör yönündeki birim vektör

c) $\hat{A} \cdot \hat{C}$

d) $\hat{B} \times \hat{C}$

e) $\hat{A} \cdot \hat{B}$

$$\textcircled{2} \hat{A} = \hat{x} + 2\hat{y} + 2\hat{z}$$

$$\hat{B} = +2\hat{x} + \hat{y} - 2\hat{z}$$

$$\hat{C} = \hat{x} - \hat{y} + \hat{z}$$

$$a) A \times (B \times C)$$

$$b) B \times (C \times A)$$

$$c) C \times (A \times B)$$

Vektör üçü çarpım

$$\Rightarrow \hat{A} \times (\hat{B} \times \hat{C}) = (\hat{A} \cdot \hat{C}) \hat{B} - (\hat{A} \cdot \hat{B}) \hat{C} \quad \text{ve}$$

$$= BAC - CAB$$

$$= \hat{B}(\hat{A} \cdot \hat{C}) - \hat{C}(\hat{A} \cdot \hat{B})$$

$$\Rightarrow (\hat{A} \times \hat{B}) \cdot (\hat{C} \times \hat{D})$$

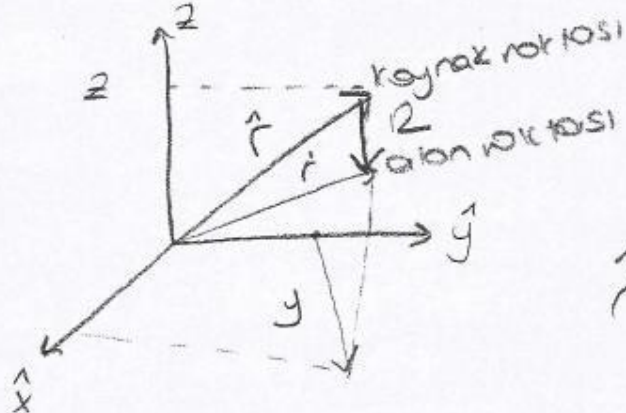
$$= (\hat{A} \cdot \hat{C})(\hat{B} \cdot \hat{D}) - (\hat{A} \cdot \hat{D})(\hat{B} \cdot \hat{C})$$

$$\Rightarrow \hat{A} \times (\hat{B} \times (\hat{C} \times \hat{D}))$$

$$= \hat{B}(\hat{A} \cdot (\hat{C} \times \hat{D})) - (\hat{A} \cdot \hat{B})(\hat{C} \times \hat{D})$$

Yer değiştirme ve Ayallık vektörleri

$$\hat{r} = x \cdot \hat{x} + y \cdot \hat{y} + z \cdot \hat{z} \quad r = \sqrt{x^2 + y^2 + z^2}$$



$$\frac{\hat{r}}{r} = \frac{x \hat{x} + y \hat{y} + z \hat{z}}{\sqrt{x^2 + y^2 + z^2}}$$

$\hat{n}$ = yer değiştirme vektörü

$$\hat{n} = \hat{r} - \hat{r}'$$

$$n = |\hat{r} - \hat{r}'|$$

$\hat{r}$ den $\hat{r}'$ ye dođru birim vektör

$$\frac{\hat{n}}{n} = \frac{\hat{r} - \hat{r}'}{|\hat{r} - \hat{r}'|} \quad \text{Kartezyen koordinat sisteminde}$$

$$\hat{n} = (x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}$$

$$n = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\frac{\hat{n}}{n} = \dots$$

ÖRNEK Kaynak vektörü $(2, 8, 7)$ den alan vektörü $(4, 6, 8)$ 'e kadar olan 2 paralel vektörün bulunuz, bu vektörün birim vektörünü hesaplayınız.

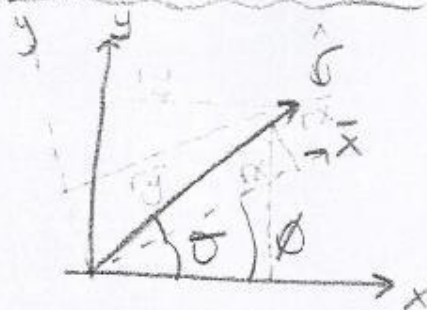
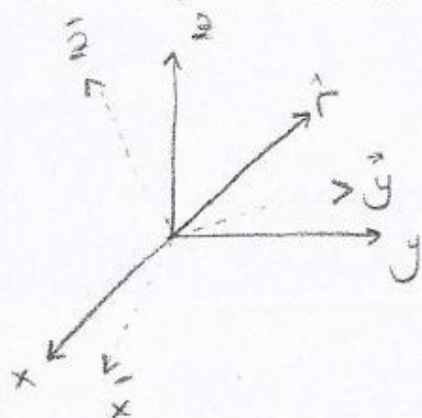
$$\hat{r} = 2\hat{x} + 8\hat{y} + 7\hat{z} \quad \hat{r}' = 4\hat{x} + 6\hat{y} + 8\hat{z} \quad \hat{n} = -2\hat{x} + 2\hat{y} - \hat{z}$$

$$n = \sqrt{(-2)^2 + 2^2 + (-1)^2} = 3$$

$$\frac{\hat{n}}{n} = -\frac{2}{3}\hat{x} + \frac{2}{3}\hat{y} - \frac{1}{3}\hat{z}$$

Kartezyen koordinat sisteminde dönüşleme

27.5.2020



θ x-y koordinat sisteminde $\hat{r}$ nin x ile yaptığı açı

$\bar{\theta}, \bar{x}, \bar{y}$ koordinat sisteminde $\hat{r}$ nin $\bar{x}$ ile yaptığı açı

$$\hat{r} = r_{\bar{x}} \cdot \hat{\bar{x}} + r_{\bar{y}} \cdot \hat{\bar{y}}$$

$$r_{\bar{x}} = r \cdot \cos \bar{\theta}$$

$$r_{\bar{y}} = r \cdot \sin \bar{\theta}$$

ϕ ile $\bar{x}$ arasındaki açı

$$\hat{r} = r_x \hat{x} + r_y \hat{y}$$

$$r_x = r \cdot \cos \theta$$

$$r_y = r \cdot \sin \theta$$

$\theta - \bar{\theta} =$ arsinin θ cinsinden verilecek

kaydırma $= \phi$

$$\bar{\theta} = \theta - \phi$$

$$r \cos(\theta - \phi) = r_{\bar{x}} = r \cdot \cos \theta \cos \phi + r \sin \theta \cdot \sin \phi$$

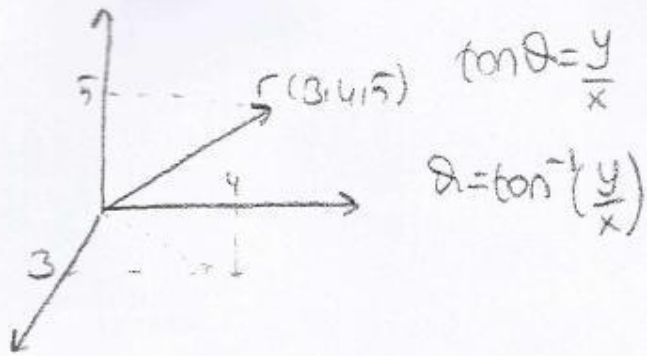
$$r \sin(\theta - \phi) = r_{\bar{y}} = r \cdot \sin \theta \cos \phi - r \cos \theta \cdot \sin \phi$$

$$\cos(\theta - \phi) = \cos \theta \cdot \cos \phi + \sin \theta \cdot \sin \phi$$

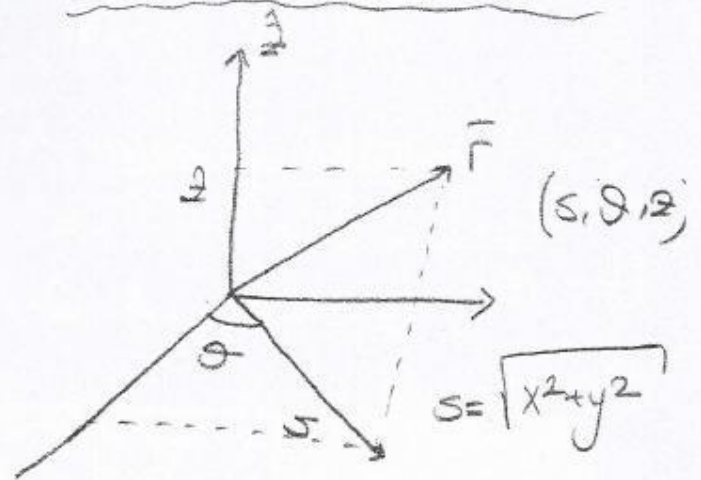
$$\sin(\theta - \phi) = \sin \theta \cdot \cos \phi - \cos \theta \cdot \sin \phi$$

$$\begin{bmatrix} r_{\bar{y}} \\ r_{\bar{x}} \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} r_y \\ r_x \end{bmatrix}$$

Kartezyen koordinat sistemi



Silindirik koordinat sistemi



$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

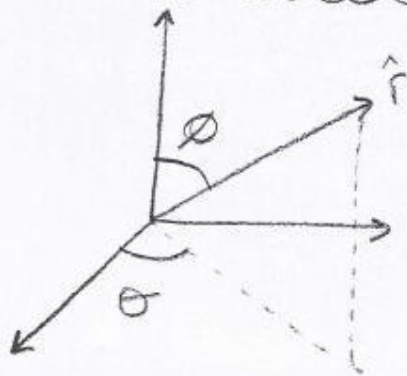
$$z = z$$

$$x = s \cos \theta$$

$$y = s \sin \theta$$

$$z = z$$

Küresel koordinat sistemi



r nin boyu sabit değer 2 ϕ deđizirler küre oluşturulur.

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

çözümü

$$\tan \theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$z = r \cos \phi$$

$$\cos \phi = \frac{z}{r}$$

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$x = r \sin \phi \cos \theta$$

$$y = r \sin \phi \sin \theta$$

$$z = r \cos \phi$$

silindirikten küreselge geçiş

$$r = \sqrt{s^2 + z^2} \quad \phi = \tan^{-1}\left(\frac{s}{z}\right)$$

Kürsesel koordinat systemen sind die 2 koordinat systeme
 gegeben

$$s = r \cdot \sin \varphi$$

$$\theta = \Theta$$

$$z = r \cdot \cos \varphi$$

	Koordinaten	sinuskoordinaten	Kürseselkoordinaten
Koordinaten		$x = s \cdot \cos \theta$ $y = s \cdot \sin \theta$ $z = z$	$x = r \cdot \sin \varphi \cdot \cos \theta$ $y = r \cdot \sin \varphi \cdot \sin \theta$ $z = r \cdot \cos \varphi$
sinuskoordinaten	$s = \sqrt{x^2 + y^2}$ $\varphi = \tan^{-1} \left(\frac{y}{x} \right)$ $z = z$		$s = r \cdot \sin \varphi$ $\theta = \theta$ $z = r \cdot \cos \varphi$
Kürseselkoordinaten	$r = \sqrt{x^2 + y^2 + z^2}$ $\varphi = \cos^{-1} \left(\frac{z}{r} \right)$ $\theta = \tan^{-1} \left(\frac{y}{x} \right)$	$r = \sqrt{s^2 + z^2}$ $\varphi = \tan^{-1} \left(\frac{s}{z} \right)$ $\theta = \theta$	