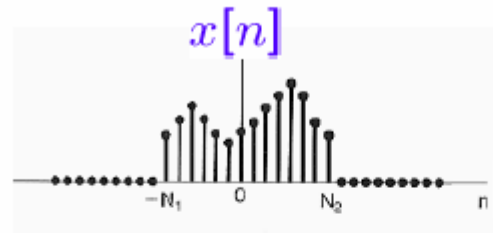


Chapter V

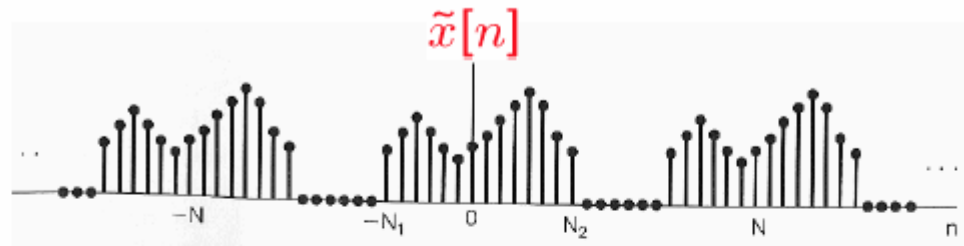
Discrete Time Fourier Transform

- Representation of Aperiodic Signals:
the Discrete-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Discrete-Time Fourier Transform
- The Convolution Property
- The Multiplication Property
- Duality
- Systems Characterized by Linear Constant-Coefficient Difference Equations

- DT Fourier Transform of an Aperiodic Signal:



an aperiodic signal



a periodic signal

$$w_0 = \frac{2\pi}{N}$$

$$\tilde{x}[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \quad \Bigg| \quad = \sum_{k=\langle N \rangle} a_k e^{jk(w_0)n}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} \tilde{x}[n] e^{-jk(2\pi/N)n} \quad \Bigg| \quad = \frac{1}{N} \sum_{n=\langle N \rangle} \tilde{x}[n] e^{-jk(w_0)n}$$

$$\Rightarrow a_k = \frac{1}{N} \sum_{n=-N_1}^{N_2} x[n] e^{-jk(w_0)n} = \frac{1}{N} \sum_{n=-\infty}^{+\infty} x[n] e^{-jk(w_0)n}$$

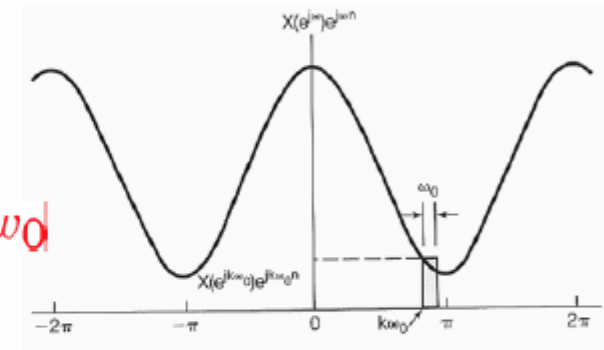
■ DT Fourier Transform of an Aperiodic Signal:

- Define $X(e^{jw})$:

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-jwn}$$

- Then,

$$a_k = \frac{1}{N} X(e^{jkw_0}) \quad w = kw_0$$



- Hence,

$$\begin{aligned} \tilde{x}[n] &= \sum_{k=\langle N \rangle} \frac{1}{N} X(e^{jkw_0}) e^{jkw_0n} & w_0 &= \frac{2\pi}{N} \\ &= \frac{1}{2\pi} \sum_{k=\langle N \rangle} X(e^{jkw_0}) e^{jkw_0n} w_0 & \frac{1}{N} &= \frac{1}{2\pi} w_0 \end{aligned}$$

▪ DT Fourier Transform of an Aperiodic Signal:

- As $N \rightarrow \infty$, $\tilde{x}[n] \rightarrow x[n]$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw}) e^{jwn} dw$$

- inverse Fourier transform eqn
- synthesis eqn

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-jwn}$$

- $X(e^{jw})$: Fourier transform of $x[n]$
spectrum
- analysis eqn

$$a_k = \frac{1}{N} X(e^{jw}) \Big|_{w=kw_0}$$

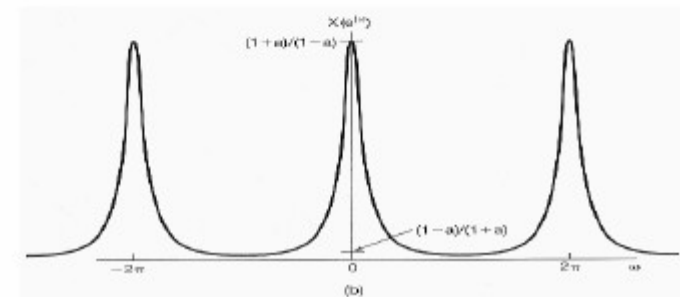
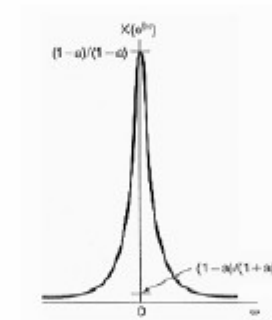
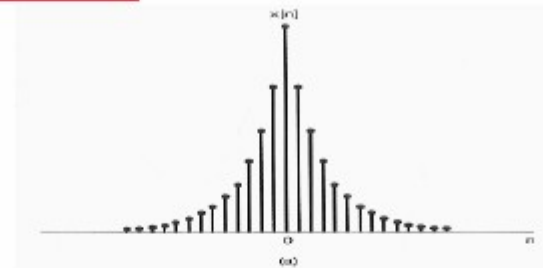
$$w_0 = \frac{2\pi}{N}$$

■ Periodicity of DT Fourier Transform:

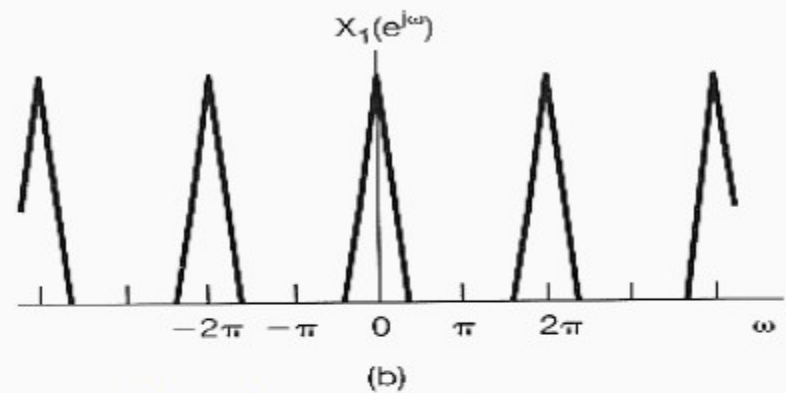
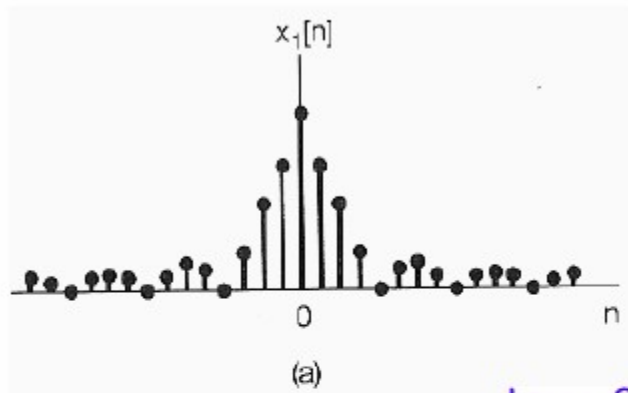
$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n}$$

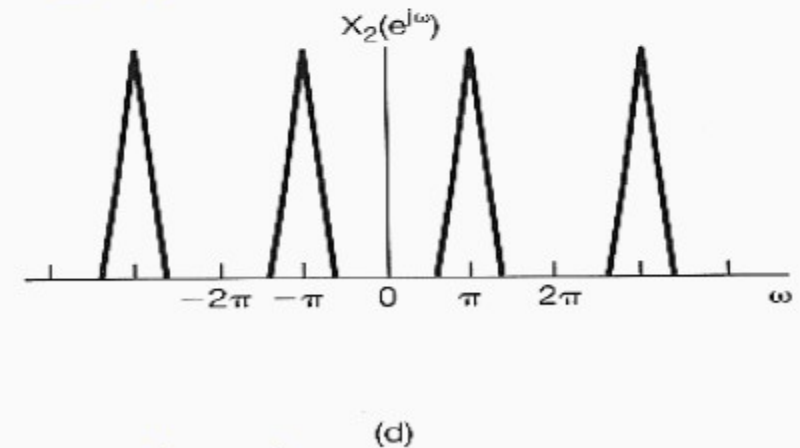
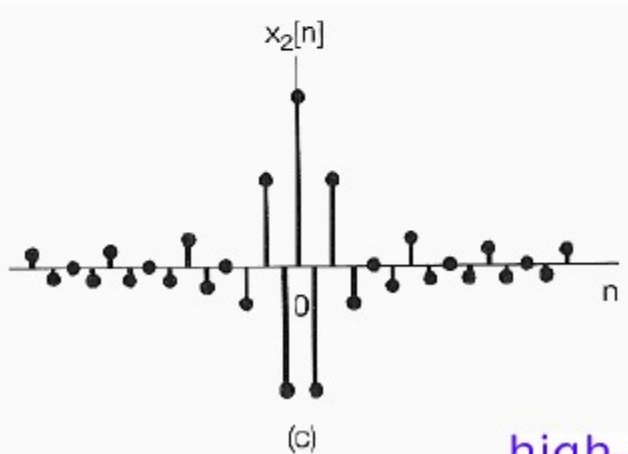
$$\begin{aligned} X(e^{j(\omega+2\pi)}) &= \sum_{n=-\infty}^{+\infty} x[n] e^{-j(\omega+2\pi)n} \\ &= \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n} e^{-j(2\pi)n} \\ &= \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n} \\ &= X(e^{j\omega}) \end{aligned}$$



■ High-Frequency & Low-Frequency Signals:



low-frequency signal



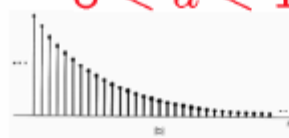
high-frequency signal

■ Example 5.1:

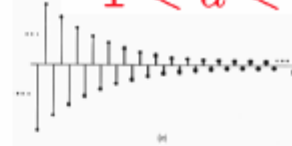
$$x[n] = a^n u[n], \quad |a| < 1$$

$$\Rightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} a^n u[n] e^{-j\omega n} = \sum_{n=0}^{\infty} (ae^{-j\omega})^n = \frac{1}{1 - ae^{-j\omega}}$$

$$0 < a < 1$$

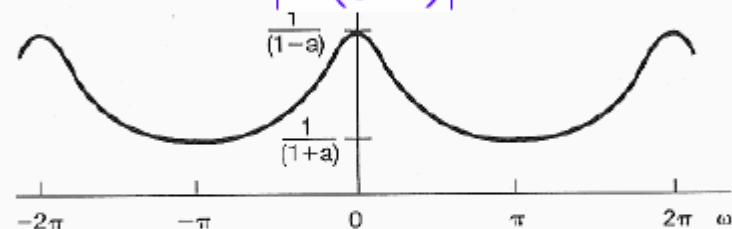


$$-1 < a < 0$$



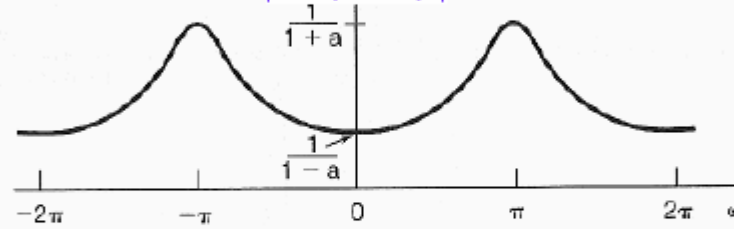
$$0 < a < 1$$

$$|X(e^{j\omega})|$$

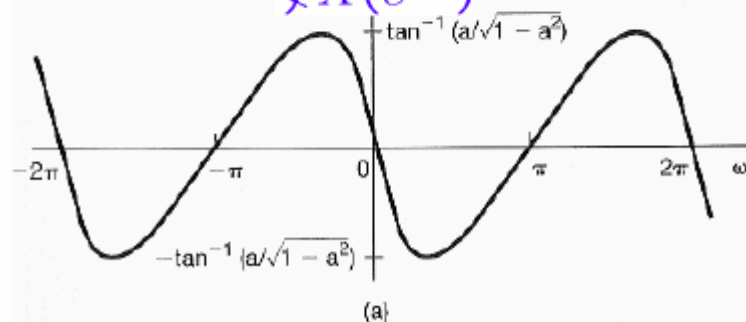


$$-1 < a < 0$$

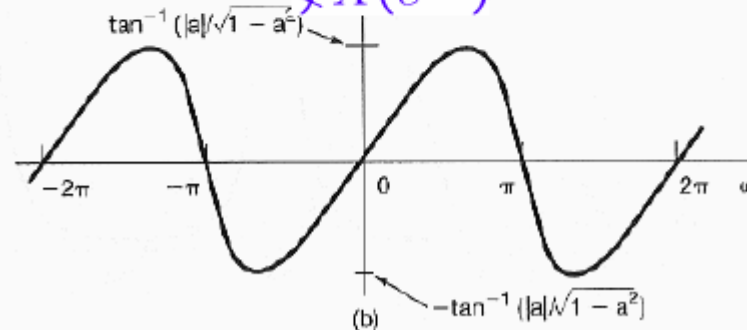
$$|X(e^{j\omega})|$$



$$\angle X(e^{j\omega})$$



$$\angle X(e^{j\omega})$$



- Example 5.2: $x[n] = a^{|n|}$, $|a| < 1$

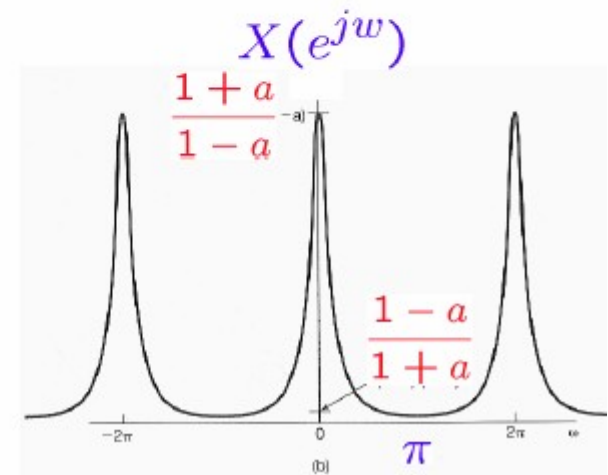
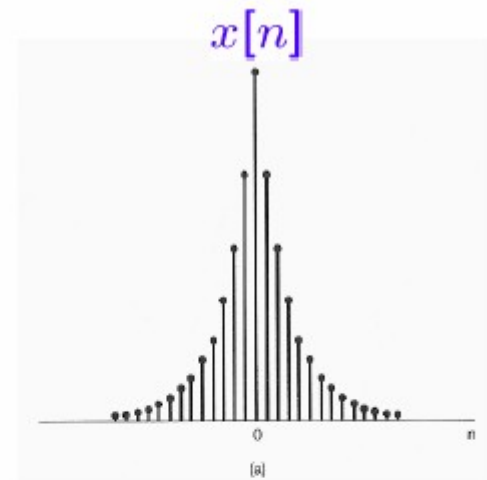
$$\Rightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} a^{|n|} e^{-j\omega n}$$

$$= \sum_{n=0}^{+\infty} a^n e^{-j\omega n} + \sum_{n=-\infty}^{-1} a^{-n} e^{-j\omega n}$$

$$= \sum_{n=0}^{+\infty} (ae^{-j\omega})^n + \sum_{m=1}^{\infty} (ae^{j\omega})^m$$

$$= \frac{1}{1 - ae^{-j\omega}} + \frac{ae^{j\omega}}{1 - ae^{j\omega}}$$

$$= \frac{1 - a^2}{1 - 2a \cos \omega + a^2} = \frac{1 - a^2}{(1 - a)^2} = \frac{1 - a^2}{(1 + a)^2}$$



■ Example 5.3:

$$x[n] = \begin{cases} 1, & |n| \leq N_1 \\ 0, & |n| > N_1 \end{cases}$$

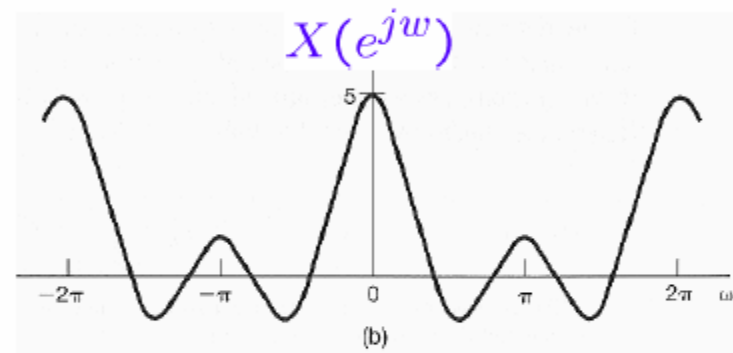
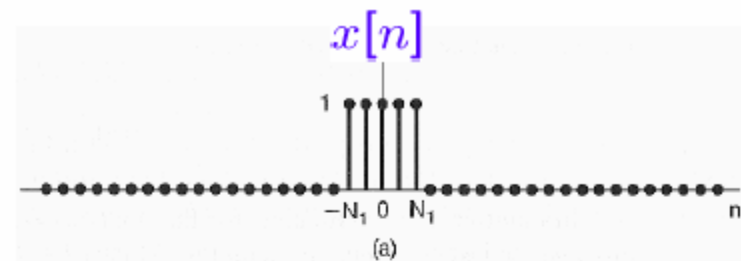
$$\Rightarrow X(e^{j\omega}) = \sum_{n=-N_1}^{N_1} e^{-j\omega n}$$

$$= e^{-j\omega(-N_1)} + \dots + e^{-j\omega(N_1)}$$

$$= e^{-j\omega(-N_1)} \left(\frac{1 - (e^{-j\omega})^{2N_1+1}}{1 - (e^{-j\omega})} \right)$$

$$= e^{j\omega(N_1)} \left(\frac{(e^{-j\omega})^{N_1+1/2} \left((e^{j\omega})^{N_1+1/2} - (e^{-j\omega})^{N_1+1/2} \right)}{(e^{-j\omega/2}) \left((e^{j\omega/2}) - (e^{-j\omega/2}) \right)} \right)$$

$$= \frac{\sin \left(\omega \left(N_1 + \frac{1}{2} \right) \right)}{\sin(\omega/2)}$$



- Convergence of DT Fourier Transform:

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw}) e^{jwn} dw$$

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-jwn}$$

- The analysis equation will converge:

- Either if $x[n]$ is absolutely summable, that is,

$$\sum_{n=-\infty}^{+\infty} |x[n]| < \infty$$

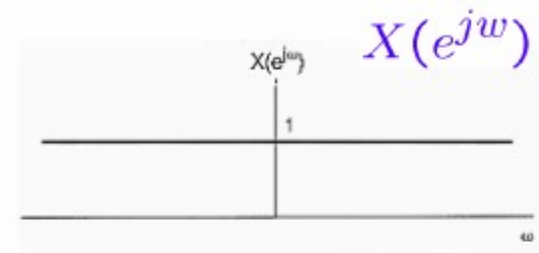
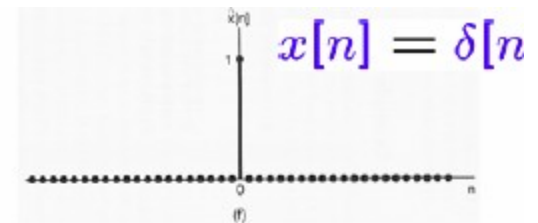
- Or if $x[n]$ has finite energy, that is,

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 < \infty$$

- Example 5.4:

$x[n] = \delta[n]$, i.e., unit impulse

$$\Rightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n} = 1$$



- Approximation

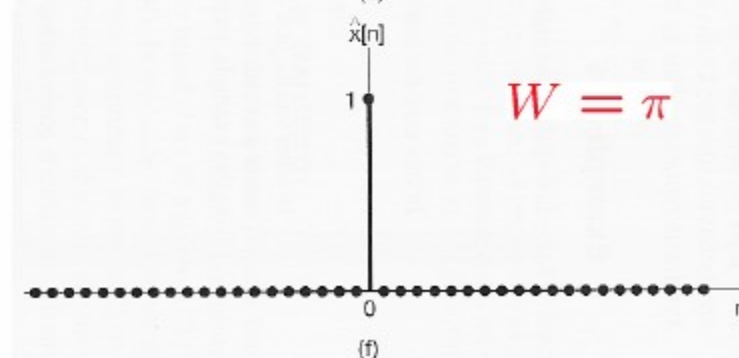
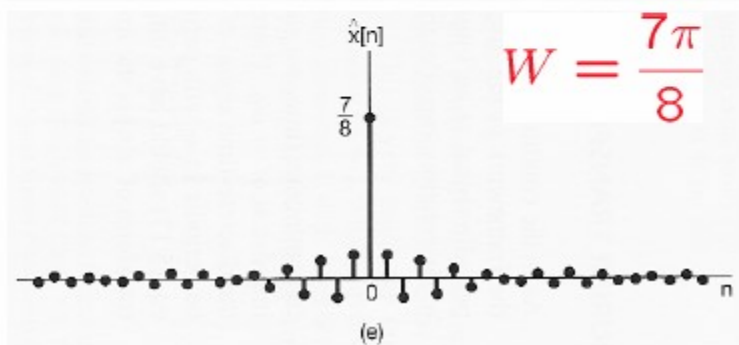
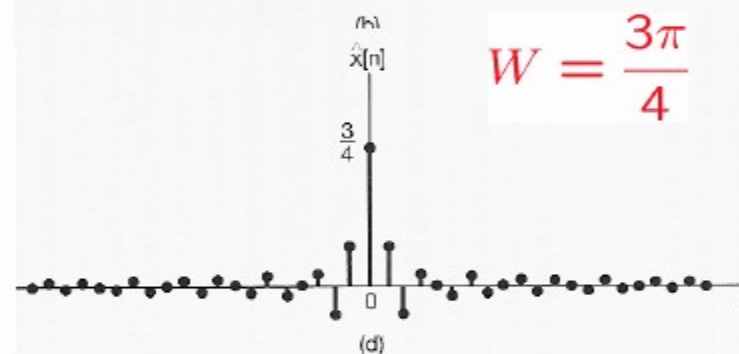
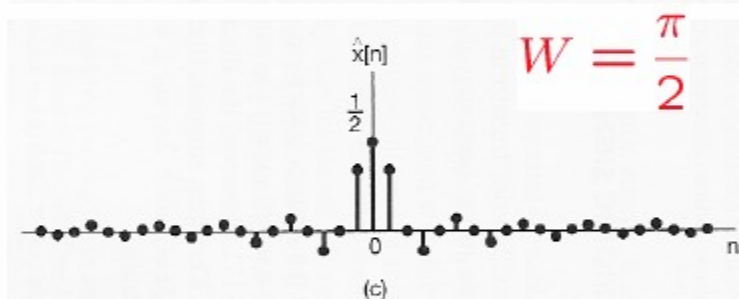
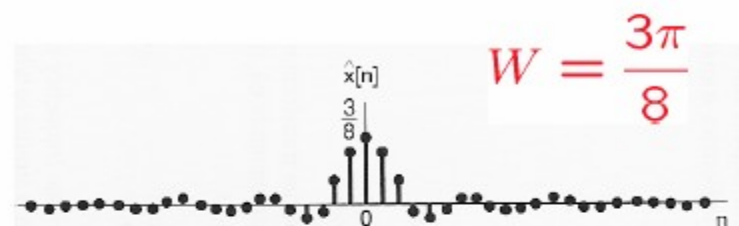
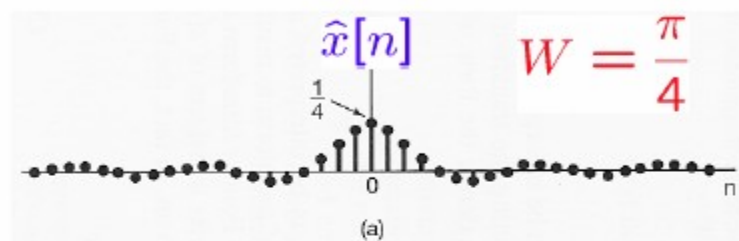
$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{2\pi} e^{j\omega n} d\omega$$

$$\hat{x}[n] = \frac{1}{2\pi} \int_{-W}^{+W} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{\sin Wn}{\pi n}$$

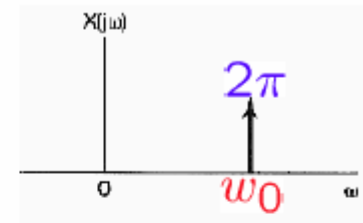
- Approximation of an Aperiodic Signal: $\hat{x}[n] = \frac{\sin Wn}{\pi n}$



- Representation of Aperiodic Signals:
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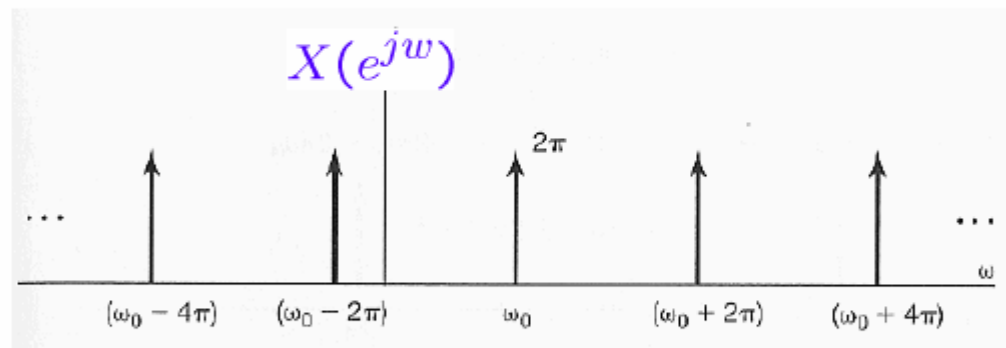
■ Fourier Transform from Fourier Series:

$$x(t) = e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} X(j\omega) = 2\pi\delta(\omega - \omega_0)$$



$$x[n] = e^{j\omega_0 n} \xleftrightarrow{\mathcal{F}} X(e^{j\omega}) = \sum_{l=-\infty}^{+\infty} 2\pi\delta(\omega - \omega_0 - 2\pi l)$$

$$\omega_0 = \frac{2\pi}{N}$$



$$\begin{aligned} \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega &= \frac{1}{2\pi} \int_{2\pi} \sum_{l=-\infty}^{+\infty} 2\pi\delta(\omega - \omega_0 - 2\pi l) e^{j\omega n} d\omega \\ &= e^{j(\omega_0 + 2\pi r)n} = e^{j\omega_0 n} \end{aligned}$$

■ Fourier Transform from Fourier Series:

- more generally,

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(\frac{2\pi}{N})n} = \sum_{k=\langle N \rangle} a_k e^{jk(w_0)n}$$

$$X(e^{jw}) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta\left(w - k\frac{2\pi}{N}\right) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(w - kw_0)$$

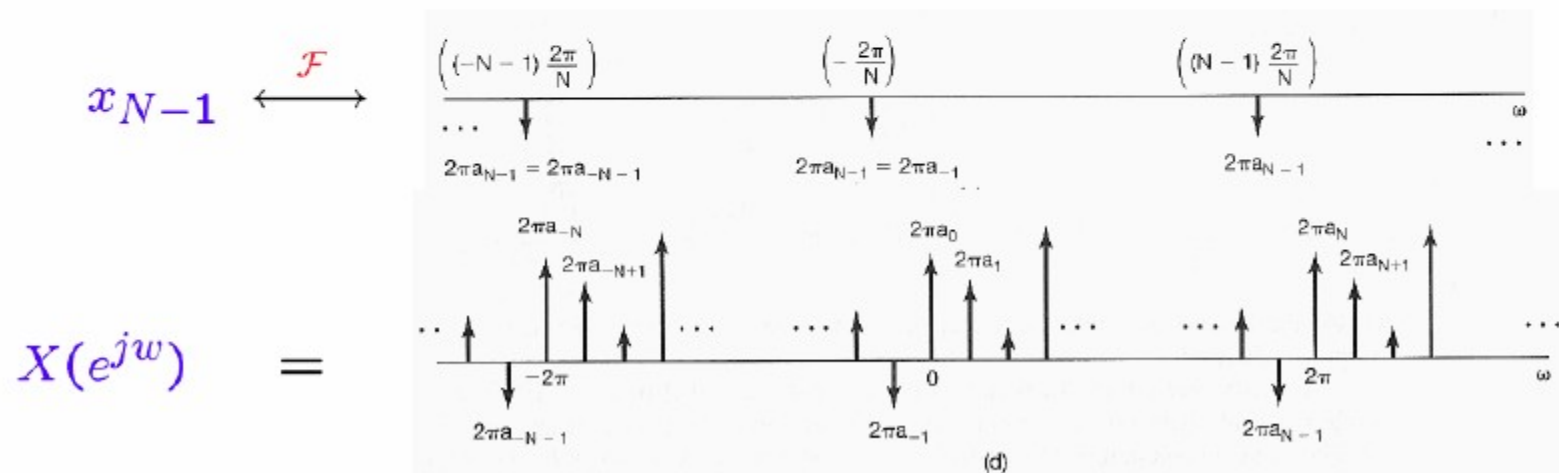
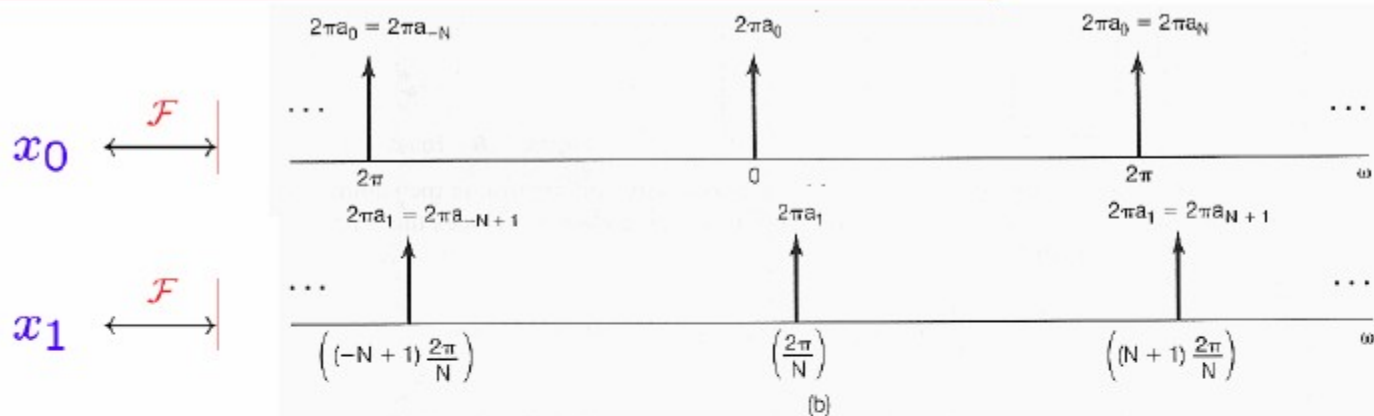
- If $k = 0, 1, \dots, N-1$

$$x[n] = a_0 + a_1 e^{j(\frac{2\pi}{N})n} + a_2 e^{j2(\frac{2\pi}{N})n} + \dots + a_{N-1} e^{j(N-1)(\frac{2\pi}{N})n}$$

$$= x_0 + x_1 + x_2 + \dots + x_{N-1}$$

a linear combination of signals
with $w_0 = 0, \frac{2\pi}{N}, \frac{4\pi}{N}, \dots, \frac{(N-1)2\pi}{N}$

■ Fourier Transform from Fourier Series:

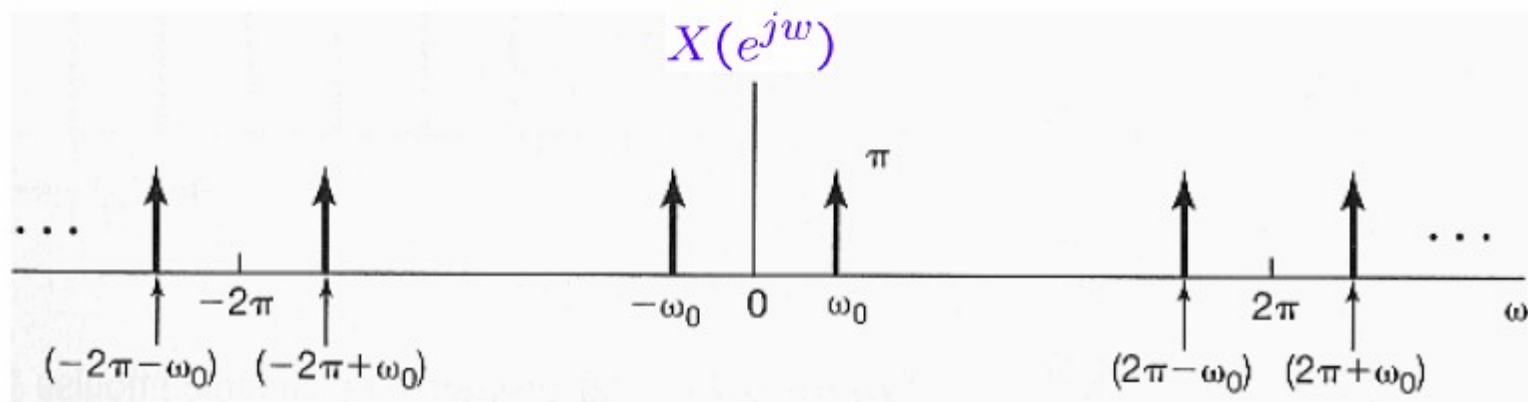


■ Example 5.5:

$$x[n] = \cos(w_0 n) = \frac{e^{jw_0 n} + e^{-jw_0 n}}{2} \quad \text{with } w_0 = \frac{2\pi}{5}$$

$$X(e^{jw}) = \sum_{l=-\infty}^{+\infty} \pi \delta\left(w - \frac{2\pi}{5} - 2\pi l\right) + \sum_{l=-\infty}^{+\infty} \pi \delta\left(w + \frac{2\pi}{5} - 2\pi l\right)$$

$$= \pi \delta\left(w - \frac{2\pi}{5}\right) + \pi \delta\left(w + \frac{2\pi}{5}\right), \quad -\pi \leq w < \pi$$



■ Example 5.6:

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n - kN]$$

choose $0 \leq n \leq N - 1$

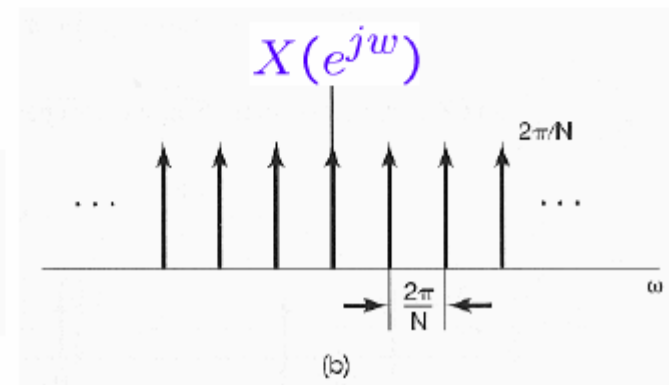
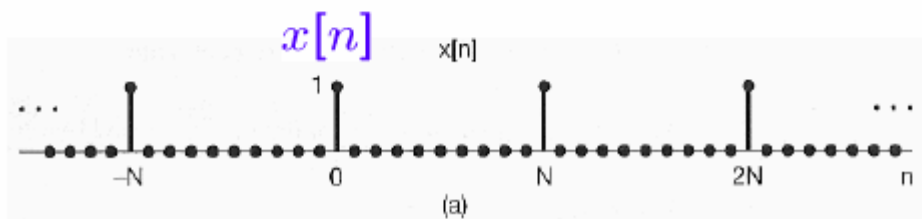
$$\Rightarrow a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} = \frac{1}{N}$$

$$\Rightarrow X(e^{jw}) = \frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta(w - k\frac{2\pi}{N})$$

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n}$$

$$X(e^{jw}) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta\left(w - \frac{2\pi k}{N}\right)$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n}$$



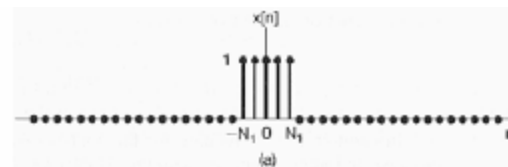
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Section	Property
5.3.2	Linearity
5.3.3	Time Shifting
5.3.3	Frequency Shifting
5.3.4	Conjugation
5.3.6	Time Reversal
5.3.7	Time Expansion
5.4	Convolution
5.5	Multiplication
5.3.5	Differencing in Time
5.3.5	Accumulation
5.3.8	Differentiation in Frequency
5.3.4	Conjugate Symmetry for Real Signals
5.3.4	Symmetry for Real and Even Signals
5.3.4	Symmetry for Real and Odd Signals
5.3.4	Even-Odd Decomposition for Real Signals
5.3.9	Parseval's Relation for Aperiodic Signals

■ Fourier Transform Pair:

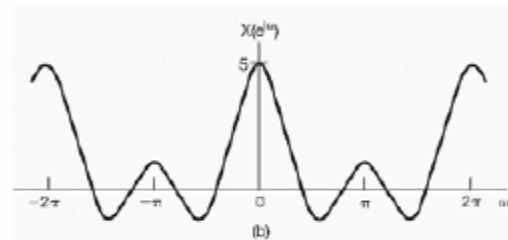
- Synthesis equation:

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$



- Analysis equation:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n}$$



- Notations:

$$X(e^{j\omega}) = \mathcal{F}\{x[n]\}$$

$$\frac{1}{1 - ae^{j\omega}} = \mathcal{F}\{a^n u[n]\}$$

$$x[n] = \mathcal{F}^{-1}\{X(e^{j\omega})\}$$

$$a^n u[n] = \mathcal{F}^{-1}\left\{\frac{1}{1 - ae^{j\omega}}\right\}$$

$$x[n] \xleftrightarrow{\mathcal{DTFT}} X(e^{j\omega})$$

$$a^n u[n] \xleftrightarrow{\mathcal{DTFT}} \frac{1}{1 - ae^{j\omega}}$$

$$|a| < 1$$

- Periodicity of DT Fourier Transform:

$$X(e^{j(w+2\pi)}) = X(e^{jw}) \quad x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw}) e^{jwn} dw$$

- Linearity:

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-jwn}$$

$$x[n] \xleftrightarrow{\mathcal{F}} X(e^{jw})$$

$$y[n] \xleftrightarrow{\mathcal{F}} Y(e^{jw})$$

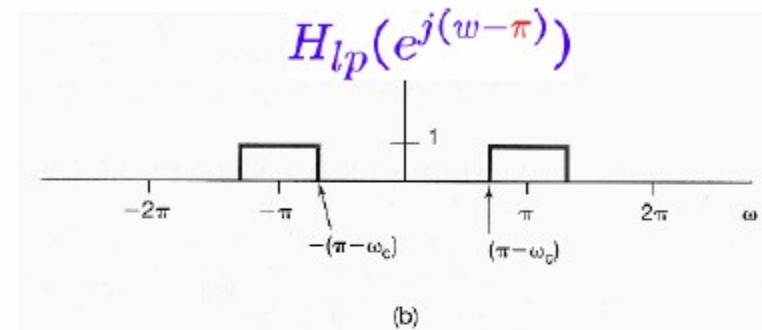
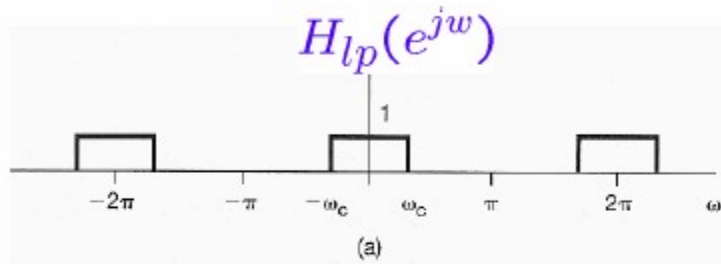
$$\Rightarrow a x[n] + b y[n] \xleftrightarrow{\mathcal{F}} a X(e^{jw}) + b Y(e^{jw})$$

- Time & Frequency Shifting:

$$\Rightarrow x[n - n_0] \xleftrightarrow{\mathcal{F}} e^{-jwn_0} X(e^{jw})$$

$$\Rightarrow e^{jw_0 n} x[n] \xleftrightarrow{\mathcal{F}} X(e^{j(w-w_0)})$$

■ Example 5.7:



$$H_{hp}(e^{j\omega}) = H_{lp}(e^{j(\omega - \pi)})$$

$$e^{j\pi n} = \cos(\pi n) + j \sin(\pi n)$$

$$\Rightarrow h_{hp}[n] = e^{j\pi n} h_{lp}[n]$$

$$= (-1)^n h_{lp}[n]$$

■ Conjugation & Conjugate Symmetry:

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw}) e^{jwn} dw$$

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-jwn}$$

$$x[n] \xleftrightarrow{\mathcal{F}} X(e^{jw}) \quad x^*[n] \xleftrightarrow{\mathcal{F}} X^*(e^{-jw})$$

- $x[n] = x^*[n] \Rightarrow X(e^{-jw}) = X^*(e^{jw})$

$x[n]$ is real $\Rightarrow X(e^{jw})$ is conjugate symmetric

- $x[n] = x^*[n] \text{ \& } x[-n] = x[n]$

$$\Rightarrow X(e^{-jw}) = X^*(e^{jw}) \text{ \& } X(e^{-jw}) = X(e^{jw})$$

$$\Rightarrow X(e^{jw}) = X^*(e^{jw})$$

$x[n]$ is real & even $\Rightarrow X(e^{jw})$ are real & even

- $x[n]$ is real & odd $\Rightarrow X(e^{jw})$ are purely imaginary & odd

- Conjugation & Conjugate Symmetry:

$$x[n] \xleftrightarrow{\mathcal{F}} X(e^{j\omega})$$

$$\mathcal{E}v\{x[n]\} \xleftrightarrow{\mathcal{F}} \mathcal{R}e\{X(e^{j\omega})\}$$

$$\mathcal{O}d\{x[n]\} \xleftrightarrow{\mathcal{F}} j \mathcal{I}m\{X(e^{j\omega})\}$$

■ Differencing & Accumulation:

$$x[n] \xleftrightarrow{\mathcal{F}} X(e^{jw})$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw}) e^{jwn} dw$$

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-jwn}$$

$$x[n] - x[n-1] \xleftrightarrow{\mathcal{F}} (1 - e^{-jw}) X(e^{jw})$$

$$X(e^{jw}) - e^{-jw} X(e^{jw})$$

$$\sum_{m=-\infty}^n x[m] \xleftrightarrow{\mathcal{F}} \frac{1}{1 - e^{-jw}} X(e^{jw}) + \pi X(e^{j0}) \sum_{k=-\infty}^{+\infty} \delta(w - 2\pi k)$$

dc or average value

$$y[n] = \sum_{m=-\infty}^n x[m] \Rightarrow y[n] - y[n-1] = x[n]$$

$$\Rightarrow (1 - e^{-jw}) Y(e^{jw}) = X(e^{jw})$$

- Differentiation in Frequency:

$$x[n] \xleftrightarrow{\mathcal{F}} X(e^{jw}) \quad \frac{d}{dw} X(e^{jw}) = \frac{d}{dw} \sum_{n=-\infty}^{+\infty} x[n] e^{-jwn}$$

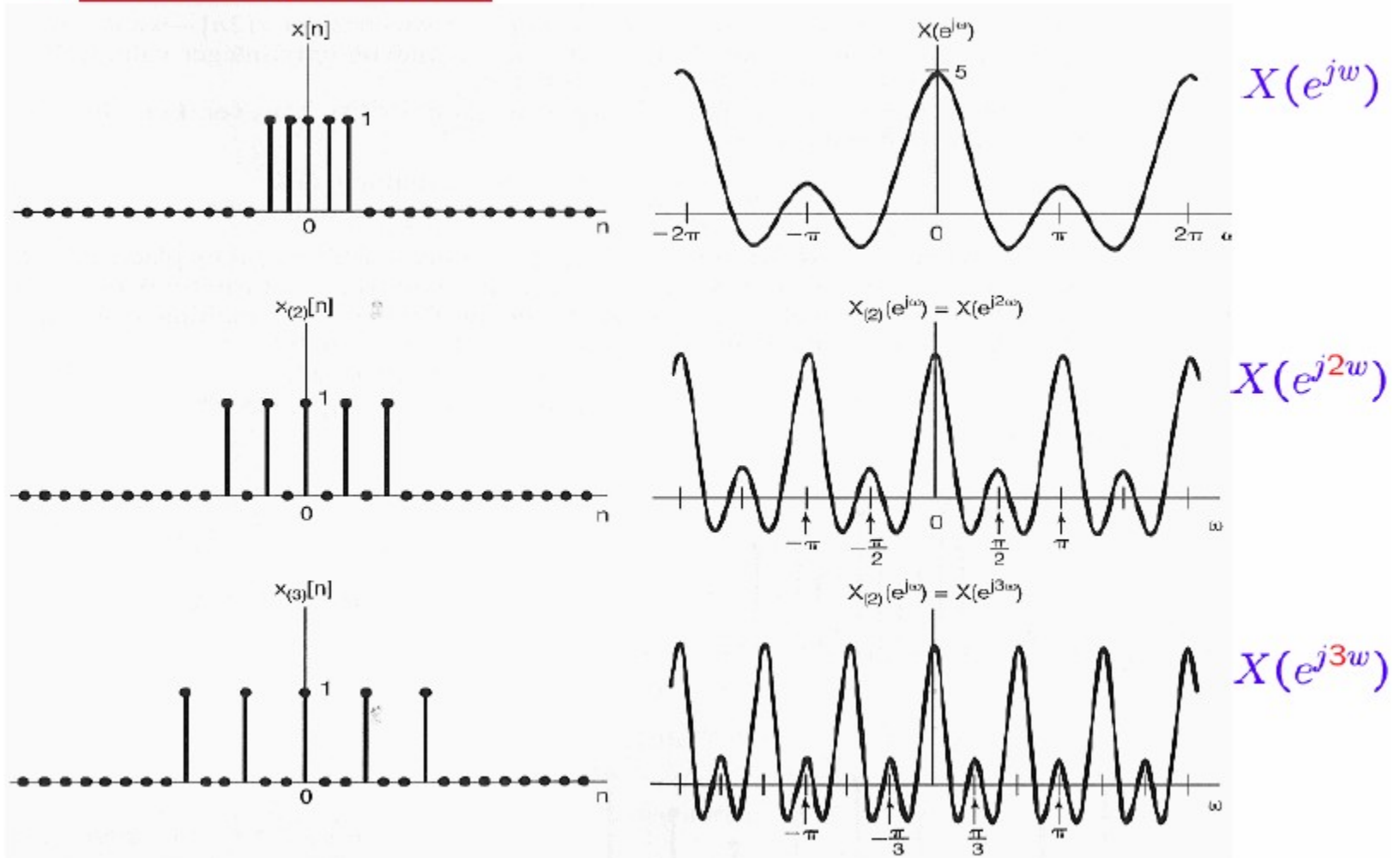
$$\begin{aligned} nx[n] &\xleftrightarrow{\mathcal{F}} j \frac{d}{dw} X(e^{jw}) \\ &= \sum_{n=-\infty}^{+\infty} (-jn) x[n] e^{-jwn} \\ &= (-j) \sum_{n=-\infty}^{+\infty} [nx[n]] e^{-jwn} \end{aligned}$$

- Time Reversal:

$$x[n] \xleftrightarrow{\mathcal{F}} X(e^{jw}) \quad X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-jwn}$$

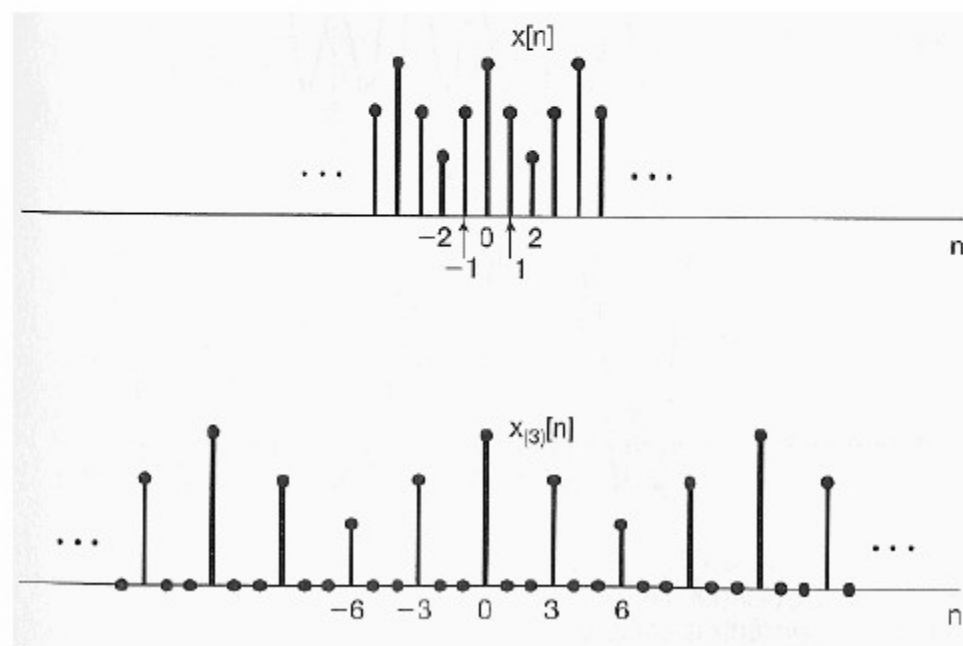
$$x[-n] \xleftrightarrow{\mathcal{F}} X(e^{-jw}) \quad X(e^{j(-w)}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j(-w)n}$$

■ Time Expansion:



- Time Expansion:

$$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n \text{ is a multiple of } k \\ 0, & \text{if } n \text{ is not a multiple of } k \end{cases}$$

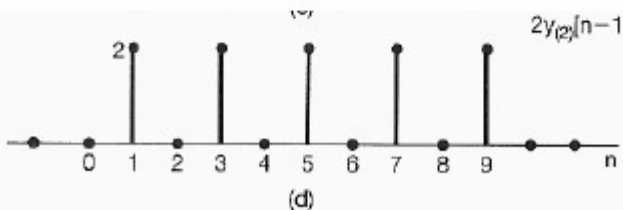
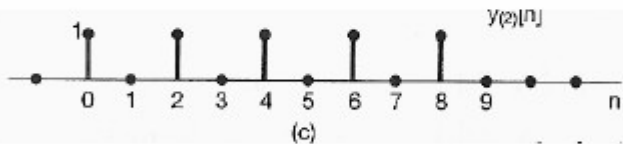
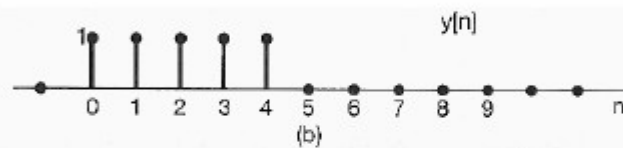
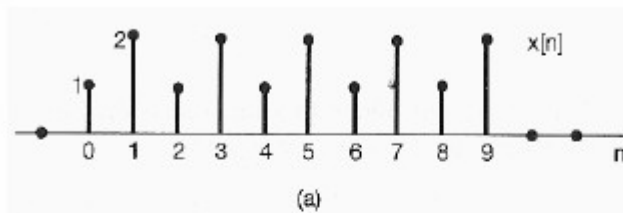


- Time Expansion:

$$\begin{aligned}
 \Rightarrow X_{(k)}(e^{jw}) &= \sum_{n=-\infty}^{+\infty} x_{(k)}[n] e^{-jwn} \\
 &= \sum_{r=-\infty}^{+\infty} x_{(k)}[rk] e^{-jw rk} \\
 &= \sum_{r=-\infty}^{+\infty} x[r] e^{-j(kw)r} \quad x_{(k)}[rk] = x[r] \\
 &= X(e^{jkw})
 \end{aligned}$$

$$x_{(k)}[n] \xleftrightarrow{\mathcal{F}} X(e^{jkw})$$

■ Example 5.9:



$$x[n] = y_{(2)}[n] + 2y_{(2)}[n-1]$$

$$Y(e^{jw}) = e^{-j2w} \frac{\sin(5w/2)}{\sin(w/2)}$$

$$y_{(2)}[n] = \begin{cases} y[n/2], & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$$

$$y_{(2)}[n] \xleftrightarrow{\mathcal{F}} e^{-j4w} \frac{\sin(5w)}{\sin(w)}$$

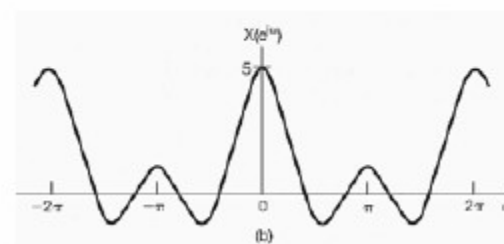
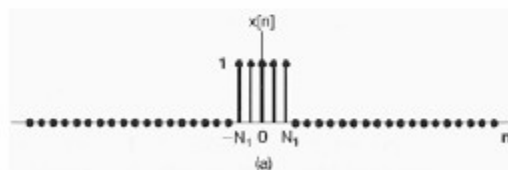
$$2y_{(2)}[n-1] \xleftrightarrow{\mathcal{F}} 2e^{-jw} e^{-j4w} \frac{\sin(5w)}{\sin(w)}$$

$$X(e^{jw}) = (1 + 2e^{-jw}) \cdot e^{-j4w} \cdot \frac{\sin(5w)}{\sin(w)}$$

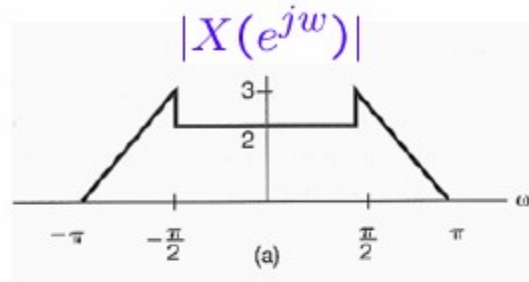
- Parseval's relation:

$$x[n] \xleftrightarrow{\mathcal{F}} X(e^{jw})$$

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |X(e^{jw})|^2 dw$$

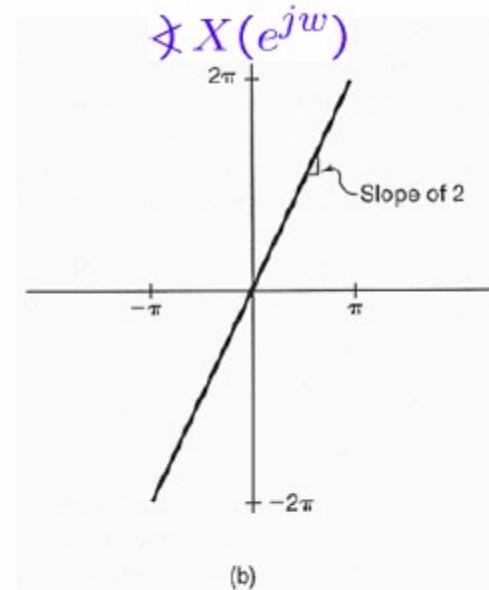


■ Example 5.10:



- $x[n]$ is **periodic**, **real**, **even**,
and/or of **finite energy**?

$\rightarrow X(e^{j\omega}) \neq 0$
 $\rightarrow$ even magnitude, odd phase
 $\rightarrow X(e^{j\omega})$ is NOT real
 $\rightarrow X(e^{j\omega})$ is finite



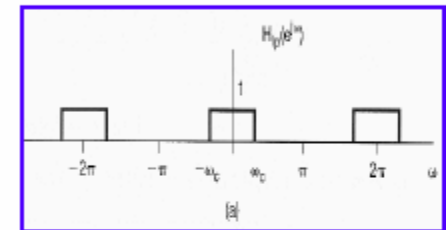
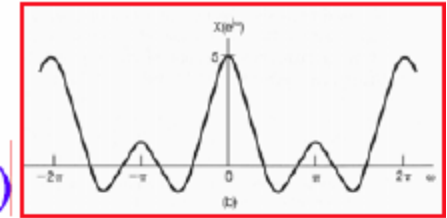
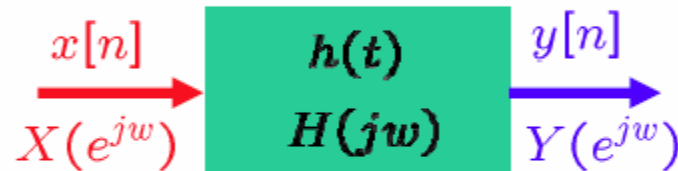
$\Rightarrow x[n]$ is **NOT** periodic
 $\Rightarrow x[n]$ is **real**
 $\Rightarrow x[n]$ is **NOT** even
 $\Rightarrow x[n]$ is **finite**

- Representation of Aperiodic Signals:
the Discrete-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Discrete-Time Fourier Transform
- The Convolution Property
- The Multiplication Property
- Duality
- Systems Characterized by Linear Constant-Coefficient Difference Equations

- Convolution Property:

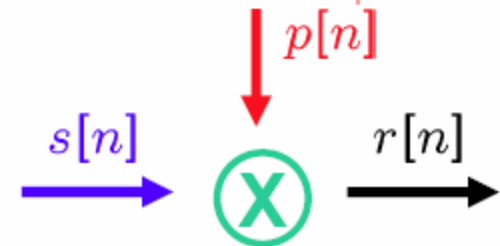
$$y[n] = x[n] * h[n] \xleftrightarrow{\mathcal{F}} Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

$$= \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

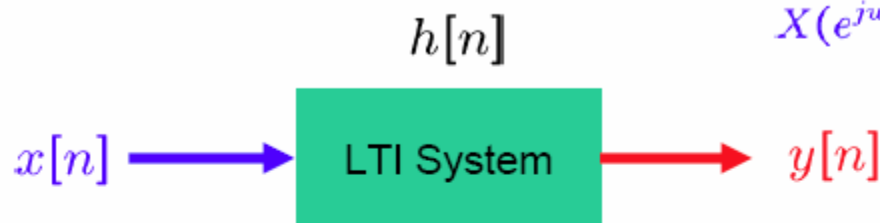


- Multiplication Property:

$$r[n] = s[n]p[n] \xleftrightarrow{\mathcal{F}} R(e^{j\omega}) = \frac{1}{2\pi} \int_{2\pi} S(e^{j\theta})P(e^{j(\omega-\theta)})d\theta$$



■ Example 5.11:



$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw}) e^{jwn} dw$$

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-jwn}$$

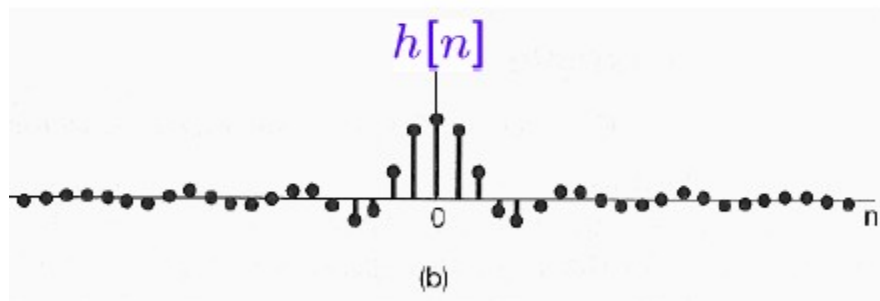
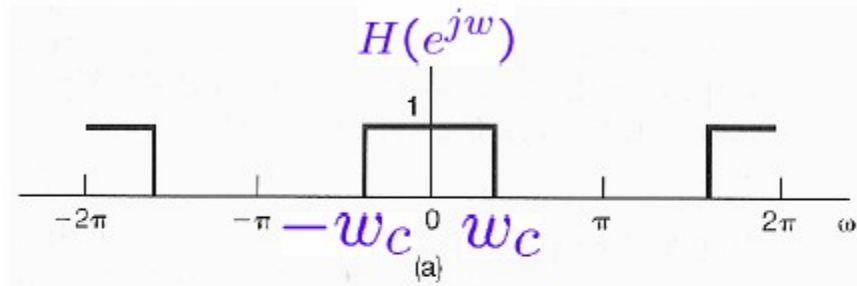
$$h[n] = \delta[n - n_0]$$

$$\Rightarrow H(e^{jw}) = \sum_{n=-\infty}^{+\infty} \delta[n - n_0] e^{-jwn} = e^{-jwn_0}$$

$$\Rightarrow Y(e^{jw}) = H(e^{jw}) X(e^{jw})$$

$$= e^{-jwn_0} X(e^{jw}) \Rightarrow y[n] = x[n - n_0]$$

■ Example 5.12:



- not causal
- oscillatory

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega$$

$$= \frac{\sin \omega_c n}{\pi n}$$

■ Example 5.13:



$$h[n] = a^n u[n], \quad |a| < 1 \quad \Rightarrow \quad H(e^{j\omega}) = \frac{1}{1 - ae^{-j\omega}}$$

$$x[n] = b^n u[n], \quad |b| < 1 \quad \Rightarrow \quad X(e^{j\omega}) = \frac{1}{1 - be^{-j\omega}}$$

$$\Rightarrow Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega})$$

$$= \frac{1}{1 - ae^{-j\omega}} \frac{1}{1 - be^{-j\omega}}$$

■ Example 5.13:

$$\text{if } a \neq b \quad Y(e^{jw}) = \left[\left(\frac{a}{a-b} \right) \frac{1}{1 - ae^{-jw}} + \left(\frac{-b}{a-b} \right) \frac{1}{1 - be^{-jw}} \right]$$

$$\Rightarrow y[n] = \left(\frac{a}{a-b} \right) a^n u[n] - \left(\frac{b}{a-b} \right) b^n u[n]$$

$$\text{if } a = b \quad Y(jw) = \left(\frac{1}{1 - ae^{-jw}} \right)^2 = \frac{j}{a} e^{jw} \frac{d}{dw} \left(\frac{1}{1 - ae^{-jw}} \right)$$

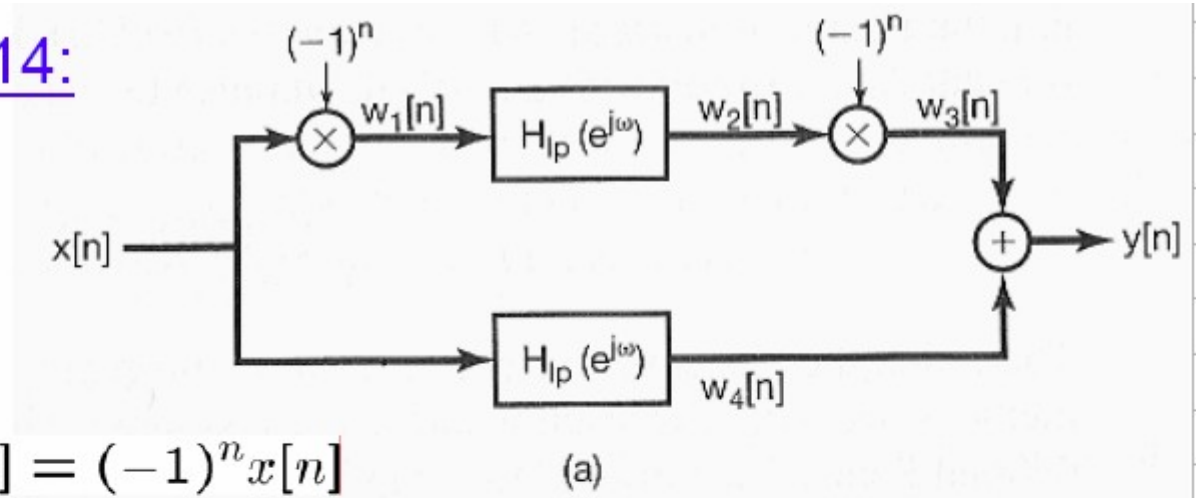
$$\text{since } a^n u[n] \xleftrightarrow{\mathcal{F}} \frac{1}{1 - ae^{-jw}}$$

$$\text{and } n a^n u[n] \xleftrightarrow{\mathcal{F}} j \frac{d}{dw} \left[\frac{1}{1 - ae^{-jw}} \right]$$

$$\text{and } (n+1) a^{n+1} u[n+1] \xleftrightarrow{\mathcal{F}} j e^{jw} \frac{d}{dw} \left[\frac{1}{1 - ae^{-jw}} \right]$$

$$\Rightarrow y[n] = (n+1) a^n u[n+1]$$

■ Example 5.14:



$$w_1[n] = e^{j\pi n} x[n] = (-1)^n x[n]$$

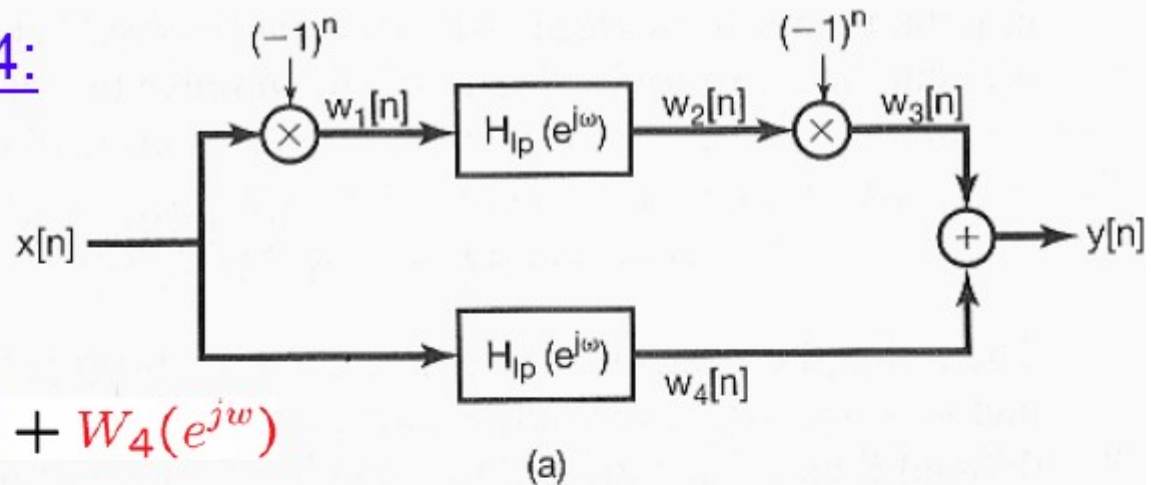
$$\Rightarrow W_1(e^{jw}) = X(e^{j(w-\pi)}) \quad W_4(e^{jw}) = H_{lp}(e^{jw}) X(e^{jw})$$

$$W_2(e^{jw}) = H_{lp}(e^{jw}) X(e^{j(w-\pi)})$$

$$w_3[n] = e^{j\pi n} w_2[n] = (-1)^n w_2[n]$$

$$\begin{aligned} \Rightarrow W_3(e^{jw}) &= W_2(e^{j(w-\pi)}) = H_{lp}(e^{j(w-\pi)}) X(e^{j(w-2\pi)}) \\ &= H_{lp}(e^{j(w-\pi)}) X(e^{jw}) \end{aligned}$$

■ Example 5.14:



$$Y(e^{jw}) = W_3(e^{jw}) + W_4(e^{jw})$$

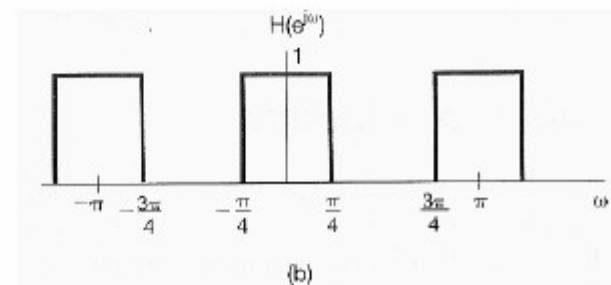
$$= H_{lp}(e^{j(w-\pi)}) X(e^{jw}) + H_{lp}(e^{jw}) X(e^{jw})$$

$$= [H_{lp}(e^{j(w-\pi)}) + H_{lp}(e^{jw})] X(e^{jw})$$

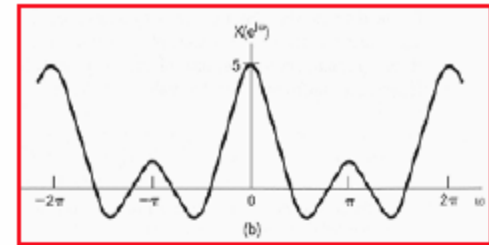
$$H(e^{jw}) = H_{lp}(e^{j(w-\pi)}) + H_{lp}(e^{jw})$$

highpass + lowpass

→ bandstop

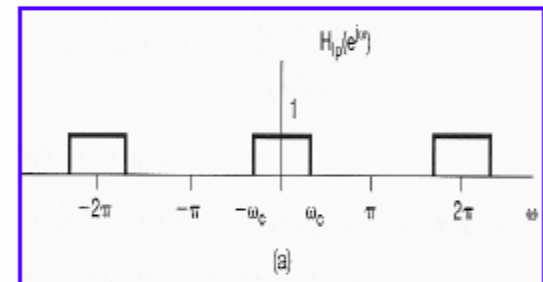


- Convolution Property:



$$y[n] = x[n] * h[n] \xleftrightarrow{\mathcal{F}} Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

$$= \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$



- Multiplication Property:

$$r[n] = s[n]p[n] \xleftrightarrow{\mathcal{F}} R(e^{j\omega}) = \frac{1}{2\pi} \int_{2\pi} S(e^{j\theta})P(e^{j(\omega-\theta)})d\theta$$

$$r[n] = s[n]p[n]$$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw}) e^{jwn} dw$$

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-jwn}$$

$$\Rightarrow R(e^{jw}) = \sum_{n=-\infty}^{+\infty} r[n] e^{-jwn}$$

$$= \sum_{n=-\infty}^{+\infty} s[n]p[n] e^{-jwn}$$

$$= \sum_{n=-\infty}^{+\infty} s[n] \left\{ \frac{1}{2\pi} \int_{2\pi} P(e^{j\theta}) e^{j\theta n} d\theta \right\} e^{-jwn}$$

$$= \frac{1}{2\pi} \int_{2\pi} P(e^{j\theta}) \left[\sum_{n=-\infty}^{+\infty} s[n] e^{-j(w-\theta)n} \right] d\theta$$

$$= \frac{1}{2\pi} \int_{2\pi} P(e^{j\theta}) S(e^{j(w-\theta)}) d\theta = \frac{1}{2\pi} \int_{2\pi} P(e^{j(w-\theta)}) S(e^{j\theta}) d\theta$$

$$y(\theta) = \int_{\Theta} x(\theta)h(\theta - \tau)d\tau$$

periodic convolution

$$y(\theta) = \int_{-\infty}^{+\infty} x(\tau)h(\theta - \tau)d\tau$$

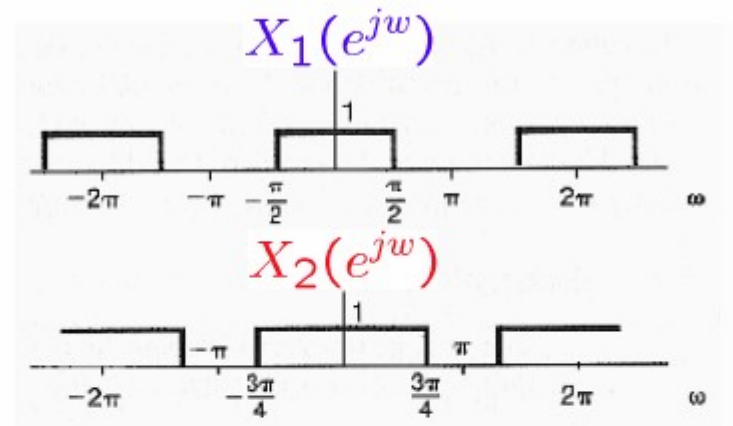
aperiodic convolution

■ Example 5.15:

$$x[n] = x_1[n]x_2[n]$$

$$x_1[n] = \frac{\sin(\frac{\pi}{2}n)}{\pi n}$$

$$x_2[n] = \frac{\sin(\frac{3\pi}{4}n)}{\pi n}$$



$$X(e^{jw}) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X_1(e^{j\theta}) X_2(e^{j(w-\theta)}) d\theta$$

$$\hat{X}_1(e^{jw}) = \begin{cases} X_1(e^{jw}), & \text{for } -\pi < w \leq \pi \\ 0, & \text{otherwise} \end{cases}$$

$$X(e^{jw}) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} \hat{X}_1(e^{j\theta}) X_2(e^{j(w-\theta)}) d\theta$$

■ Example 5.15:

$$\begin{aligned}
 X(e^{j\omega}) &= \frac{1}{2\pi} \int_{-\pi}^{+\pi} \hat{X}_1(e^{j\theta}) X_2(e^{j(\omega-\theta)}) d\theta \\
 &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{X}_1(e^{j\theta}) X_2(e^{j(\omega-\theta)}) d\theta
 \end{aligned}$$

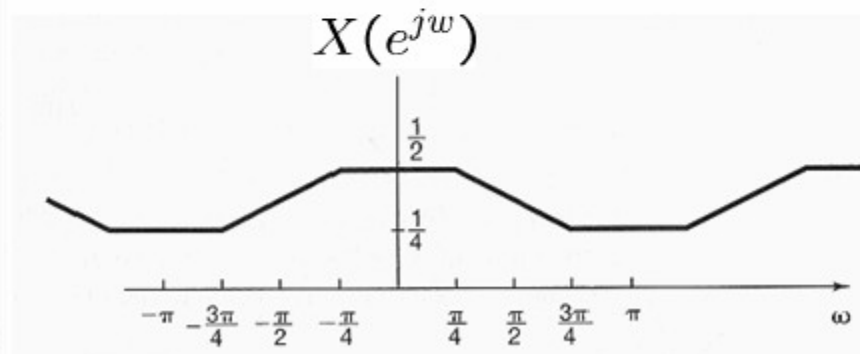
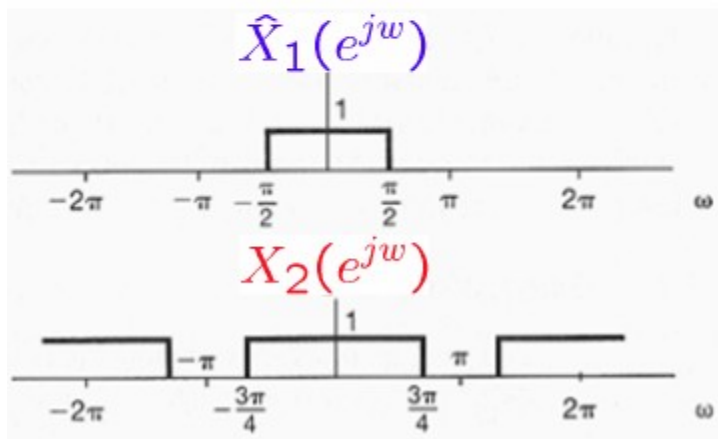


TABLE 5.1 PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Section	Property	Aperiodic Signal	Fourier Transform
		$x[n]$	$X(e^{j\omega})$ periodic with
		$y[n]$	$Y(e^{j\omega})$ period 2π
5.3.2	Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
5.3.3	Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0} X(e^{j\omega})$
5.3.3	Frequency Shifting	$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$
5.3.4	Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
5.3.6	Time Reversal	$x[-n]$	$X(e^{-j\omega})$
5.3.7	Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{j\omega})$
5.4	Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
5.5	Multiplication	$x[n]y[n]$	$\frac{1}{2\pi} \int_{2\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)})d\theta$
5.3.5	Differencing in Time	$x[n] - x[n - 1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
5.3.5	Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - e^{-j\omega}} X(e^{j\omega})$ $+ \pi X(e^{j0}) \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$
5.3.8	Differentiation in Frequency	$nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$
5.3.4	Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \Re\{X(e^{j\omega})\} = \Re\{X(e^{-j\omega})\} \\ \Im\{X(e^{j\omega})\} = -\Im\{X(e^{-j\omega})\} \\ X(e^{j\omega}) = X(e^{-j\omega}) \\ \angle X(e^{j\omega}) = -\angle X(e^{-j\omega}) \end{cases}$
5.3.4	Symmetry for Real, Even Signals	$x[n]$ real and even	$X(e^{j\omega})$ real and even
5.3.4	Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
5.3.4	Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}\{x[n]\} \quad [x[n] \text{ real}]$ $x_o[n] = \mathcal{O}\{x[n]\} \quad [x[n] \text{ real}]$	$\Re\{X(e^{j\omega})\}$ $j\Im\{X(e^{j\omega})\}$
5.3.9	Parseval's Relation for Aperiodic Signals		$\sum_{n=-\infty}^{+\infty} x[n] ^2 = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) ^2 d\omega$

TABLE 5.2 BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS

Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\sum_{k \in \langle N \rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \dots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & N_1 < n \leq N/2 \end{cases}$ and $x[n + N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]}, \quad k \neq 0, \pm N, \pm 2N, \dots$ $a_k = \frac{2N_1 + 1}{N}, \quad k = 0, \pm N, \pm 2N, \dots$

$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & N_1 < n \leq N/2 \end{cases}$ and $x[n+N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]}, \quad k \neq 0, \pm N, \pm 2N, \dots$ $a_k = \frac{2N_1 + 1}{N}, \quad k = 0, \pm N, \pm 2N, \dots$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N} \text{ for all } k$
$a^n u[n], \quad a < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & n > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \text{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \leq \omega \leq W \\ 0, & W < \omega \leq \pi \end{cases}$ $X(\omega)$ periodic with period 2π	—
$\delta[n]$	1	—
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi \delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n+1)a^n u[n], \quad a < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	—
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n], \quad a < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	—

- Representation of Aperiodic Signals:
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■ DT Fourier Series Pair of Periodic Signals:

- $x[n] \xleftrightarrow{\mathcal{FS}} a_k$: DT Fourier series pair

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n}$$

$$\text{IF } f[k] = \frac{1}{N} \sum_{n=\langle N \rangle} g[n] e^{-jk(2\pi/N)n} \quad g[n] \xleftrightarrow{\mathcal{FS}} f[k]$$

$$f[n] = \sum_{k=\langle N \rangle} \frac{1}{N} g[-k] e^{jk(2\pi/N)n} \quad f[n] \xleftrightarrow{\mathcal{FS}} \frac{1}{N} g[-k]$$

$$\text{LET } k = n, n = -k \quad a_k := \frac{1}{N} x[-n]$$

- Duality in DT Fourier Series:

$$x[n] \xleftrightarrow{\mathcal{FS}} a_k$$

$$x[n-n_0] \xleftrightarrow{\mathcal{FS}} a_k e^{-jk(2\pi/N)n_0}$$

$$e^{+jm(2\pi/N)n} x[n] \xleftrightarrow{\mathcal{FS}} a_{k-m}$$

$$\sum_{r=\langle N \rangle} x[r] y[n-r] \xleftrightarrow{\mathcal{FS}} N a_k b_k$$

$$x[n] y[n] \xleftrightarrow{\mathcal{FS}} \sum_{l=\langle N \rangle} a_l b_{k-l}$$

- Duality between DT-FT & CT-FS:

$$x[n] \xleftrightarrow{\text{DTFT}} X(e^{j\omega})$$

$$x(t) \xleftrightarrow{\text{FS}} a_k$$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n}$$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

TABLE 5.3 SUMMARY OF FOURIER SERIES AND TRANSFORM EXPRESSIONS

	Continuous time			Discrete time	
	Time domain	Frequency domain		Time domain	Frequency domain
Fourier Series	$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$ continuous time periodic in time	$a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$ discrete frequency aperiodic in frequency		$x[n] = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/N)n}$ discrete time periodic in time	$a_k = \frac{1}{N} \sum_{n=-\infty}^{+\infty} x[n] e^{-jk(2\pi/N)n}$ discrete frequency periodic in frequency
Fourier Transform	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega$ continuous time aperiodic in time	$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$ continuous frequency aperiodic in frequency		$x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{j\omega}) e^{j\omega n} d\omega$ discrete time aperiodic in time	$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n}$ continuous frequency periodic in frequency

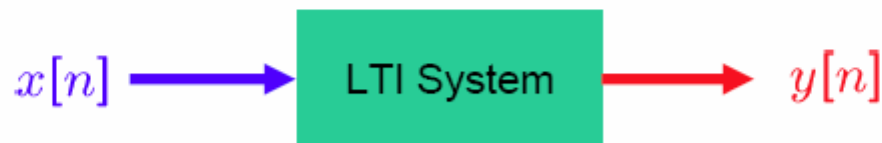
- Representation of Aperiodic Signals:
the Discrete-Time Fourier Transform
- The Fourier Transform for Periodic Signals
- Properties of Discrete-Time Fourier Transform
- The Convolution Property
- The Multiplication Property
- Duality
- Systems Characterized by Linear Constant-Coefficient Difference Equations

- A useful class of DT LTI systems:

$$a_0 y[n] + a_1 y[n-1] + \cdots + a_{N-1} y[n-N+1] + a_N y[n-N]$$

$$= b_0 x[n] + b_1 x[n-1] + \cdots + b_{M-1} x[n-M+1] + b_M x[n-M]$$

$$\sum_{k=0}^N a_k y[n-k] = \sum_{k=0}^M b_k x[n-k]$$



$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) \quad H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

$$x[n - n_0] \xleftrightarrow{\mathcal{F}} e^{-j\omega n_0} X(e^{j\omega})$$

$$\mathcal{F} \left\{ \sum_{k=0}^N a_k y[n - k] \right\} = \mathcal{F} \left\{ \sum_{k=0}^M b_k x[n - k] \right\}$$

$$\sum_{k=0}^N a_k \mathcal{F} \left\{ y[n - k] \right\} = \sum_{k=0}^M b_k \mathcal{F} \left\{ x[n - k] \right\}$$

$$\sum_{k=0}^N a_k e^{-jk\omega} Y(e^{j\omega}) = \sum_{k=0}^M b_k e^{-jk\omega} X(e^{j\omega})$$

$$\begin{aligned} \Rightarrow H(e^{j\omega}) &= \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{k=0}^M b_k e^{-jk\omega}}{\sum_{k=0}^N a_k e^{-jk\omega}} \\ &= \frac{b_0 + b_1 e^{-j\omega} + \dots + b_M e^{-jM\omega}}{a_0 + a_1 e^{-j\omega} + \dots + a_N e^{-jN\omega}} \end{aligned}$$

■ Examples 5.18 & 5.19:



$$|a| < 1$$

$$y[n] - ay[n-1] = x[n] \quad \Rightarrow \quad H(e^{jw}) = \frac{1}{1 - ae^{-jw}}$$

$$Y(\cdot) - e^{-jw}Y(\cdot) \Rightarrow h[n] = a^n u[n]$$

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$$

$$\Rightarrow H(e^{jw}) = \frac{2}{1 - \frac{3}{4}e^{-jw} + \frac{1}{8}e^{-j2w}} = \frac{2}{(1 - \frac{1}{2}e^{-jw})(1 - \frac{1}{4}e^{-jw})}$$

$$= \frac{4}{(1 - \frac{1}{2}e^{-jw})} - \frac{2}{(1 - \frac{1}{4}e^{-jw})}$$

$$\Rightarrow h[n] = 4\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n]$$

■ Example 5.20:

$$h[n] \xleftrightarrow{\mathcal{F}} H(e^{j\omega}) = \frac{2}{(1 - \frac{1}{2}e^{-j\omega})(1 - \frac{1}{4}e^{-j\omega})}$$

$$x[n] = \left(\frac{1}{4}\right)^n u[n] \xrightarrow{\text{LTI System}} y[n] = ???$$

$$= x[n] * h[n]$$

$$\Rightarrow Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

$$= \left[\frac{1}{1 - \frac{1}{4}e^{-j\omega}} \right] \left[\frac{2}{(1 - \frac{1}{2}e^{-j\omega})(1 - \frac{1}{4}e^{-j\omega})} \right]$$

$$= \frac{2}{(1 - \frac{1}{2}e^{-j\omega})(1 - \frac{1}{4}e^{-j\omega})}$$

$$= \frac{8}{(1 - \frac{1}{2}e^{-j\omega})} - \frac{4}{(1 - \frac{1}{4}e^{-j\omega})} - \frac{2}{(1 - \frac{1}{4}e^{-j\omega})}$$

$$\Rightarrow y[n] = \left\{ 8 \left(\frac{1}{2}\right)^n - 4 \left(\frac{1}{4}\right)^n - 2(n+1) \left(\frac{1}{4}\right)^n \right\} u[n]$$

Signals & Systems [\(Chap 1\)](#)

LTI & Convolution [\(Chap 2\)](#)

Bounded/Convergent

Periodic

FS

[\(Chap 3\)](#)

– CT

– DT

Aperiodic

FT

– CT

[\(Chap 4\)](#)

– DT

[\(Chap 5\)](#)

Unbounded/Non-convergent

LT

– CT

[\(Chap 9\)](#)

zT

– DT

[\(Chap 10\)](#)

Time-Frequency [\(Chap 6\)](#)

Communication [\(Chap 8\)](#)

CT-DT [\(Chap 7\)](#)

Control [\(Chap 11\)](#)