

Chapter IV

Fourier Integral

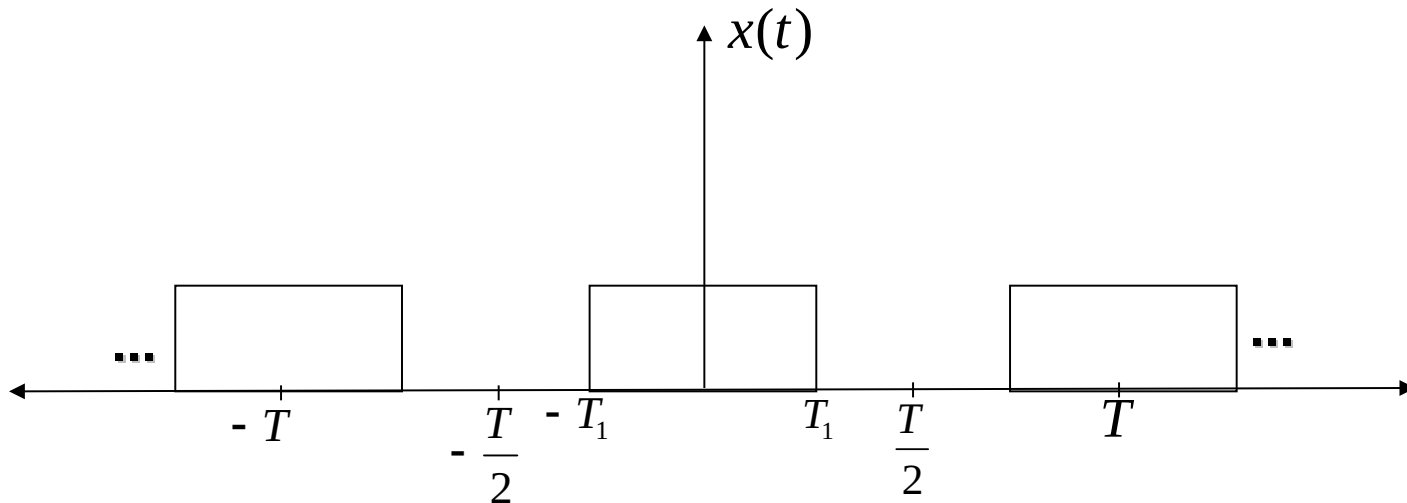
Continuous–Time Fourier Transform

- So far, we have seen periodic signals and their representation in terms of linear combination of complex exponentials.
- Practical meaning:
 - superposition of harmonically related complex exponentials.
- How about aperiodic signals?

Fourier Transform Representation of Aperiodic Signals

- (An aperiodic signal is a periodic signal with infinite period.)
- Let us consider *continuous-time periodic square wave*

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$



Fourier Transform Representation of Aperiodic Signals

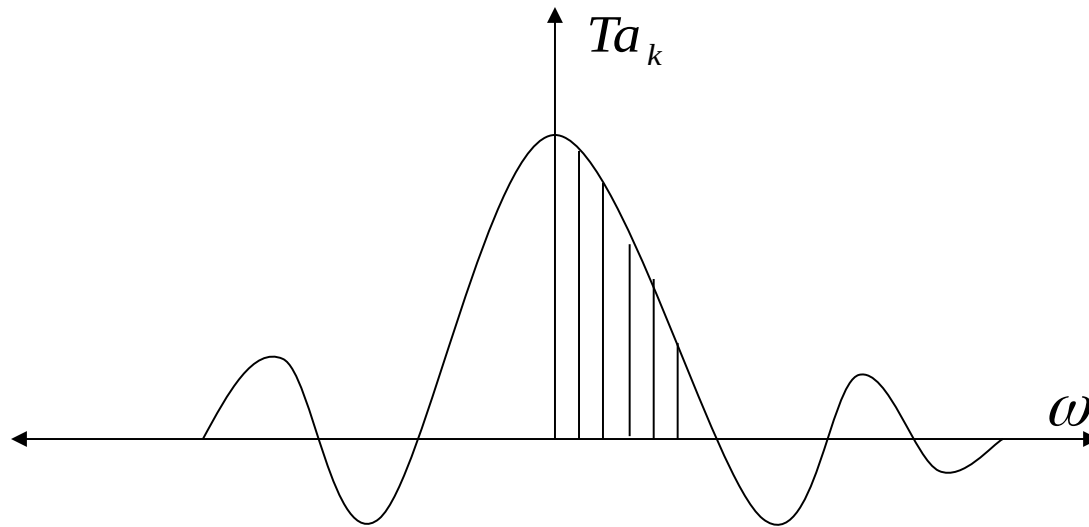
- Recall that the FS coefficients are:

$$a_k = \frac{2 \sin(k \omega_0 T_1)}{k \omega_0 T}, \quad \text{for } \omega_0 = \frac{2\pi}{T}$$

- Let us look at it as:

$$T a_k = \frac{2 \sin \omega T_1}{\omega} \Big|_{\omega=k\omega_0} : \text{samples of an envelope function}$$

Fourier Transform Representation of Aperiodic Signals



as $T \uparrow$, $\omega_0 \downarrow$, $\rightarrow$ closer samples, faster rate!
as $T \rightarrow \infty$, periodic square wave $\rightarrow$ rectangular pulse (aperiodic signal)
FS coefficients $\times T \rightarrow$ envelope itself

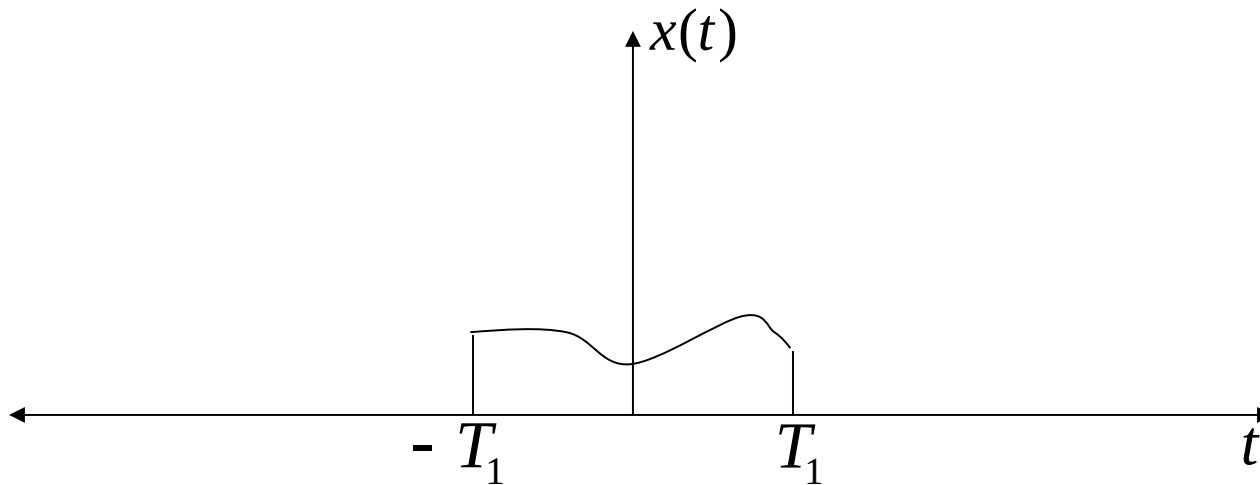
$\rightarrow$ Think of aperiodic signal as the limit of a periodic signal

Fourier Transform Representation of Aperiodic Signals

→ Examine the limiting behaviour of FS representation of this signal

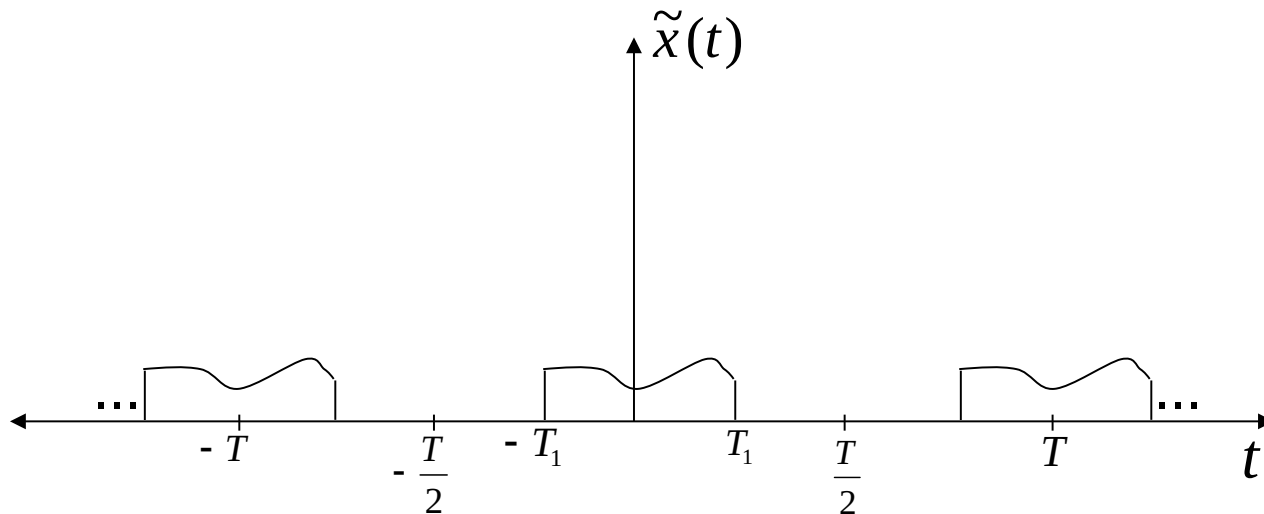
→ Consider a signal $x(t)$ that is of finite duration,

$$x(t) = 0, |t| > T_1$$



Fourier Transform Representation of Aperiodic Signals

- We can construct a periodic signal $\tilde{x}(t)$ out of $x(t)$ with period T
- For $T \rightarrow \infty$, $\tilde{x}(t) = x(t)$.



$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}, \quad a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t} dt, \quad \text{for } \omega_0 = \frac{2\pi}{T}$$

FS representation of periodic signal

Fourier Transform Representation of Aperiodic Signals

Since $\tilde{x}(t) = x(t)$ for $|t| < \frac{T}{2}$ and $x(t) = 0$ for $|t| > \frac{T}{2}$

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_{-\infty}^{\infty} x(t) e^{-jk\omega_0 t} dt$$

(Recall Ta_k and envelope case)

Define $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ as the envelope of Ta_k

$$\Rightarrow a_k = \frac{1}{T} X(k\omega_0) \quad (\text{for } \omega = k\omega_0)$$

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} X(k\omega_0) e^{jk\omega_0 t}$$

Fourier Transform Representation of Aperiodic Signals

$$\text{For } \frac{2\pi}{T} = \omega_0 \quad \Rightarrow \quad \tilde{x}(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} X(k\omega_0) \cdot e^{jk\omega_0 t} \omega_0$$

$$\text{As } T \rightarrow \infty, \quad \omega_0 \rightarrow 0, \quad k\omega_0 \rightarrow \omega, \quad \Sigma \rightarrow \int \quad \tilde{x} \rightarrow x(t)$$

Hence,

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

: inverse Fourier transform



Fourier Transform pair

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

*: Fourier transform
(spectrum)*

→ *information needed for describing $x(t)$ as a linear combination of sinusoids*

Convergence of Fourier Transform

Dirichlet conditions guarantee that $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$ at any t except at discontinuities (similar to periodic case).

i) $x(t)$ is absolutely integrable,
$$\int_{-\infty}^{\infty} |x(t)| dt < \infty$$

ii) $x(t)$ has a finite number of maxima and minima within any finite interval

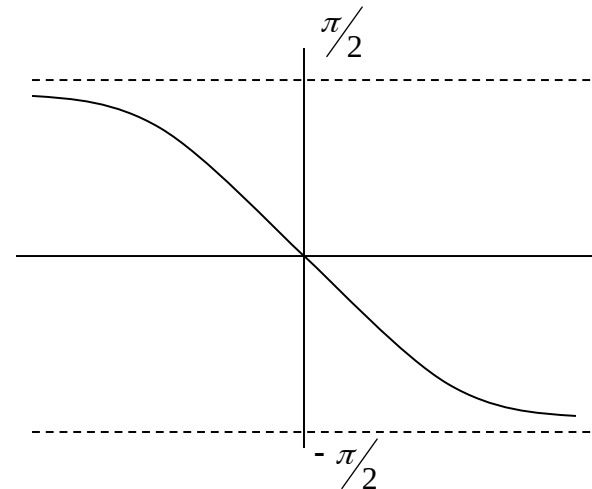
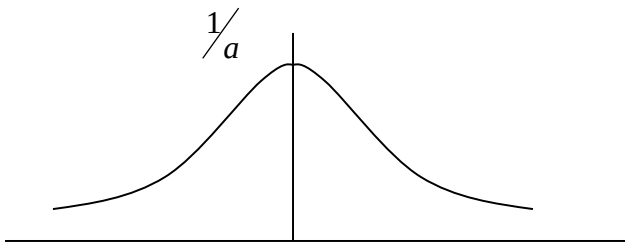
iii) $x(t)$ has a finite number of discontinuities within any finite interval. Further more each of these discontinuities must be finite.

Example

Consider $x(t) = e^{-at}u(t)$, $a > 0$

$$X(\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt = - \frac{1}{a + j\omega} e^{-(a+j\omega)t} \bigg|_0^{\infty} = \frac{1}{a + j\omega}, \quad a > 0$$

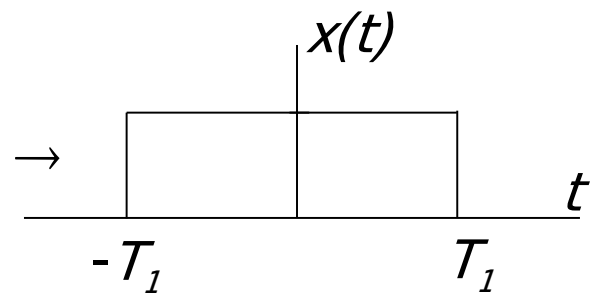
$$|X(\omega)| = \frac{1}{\sqrt{a^2 + \omega^2}}, \quad \angle X(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$



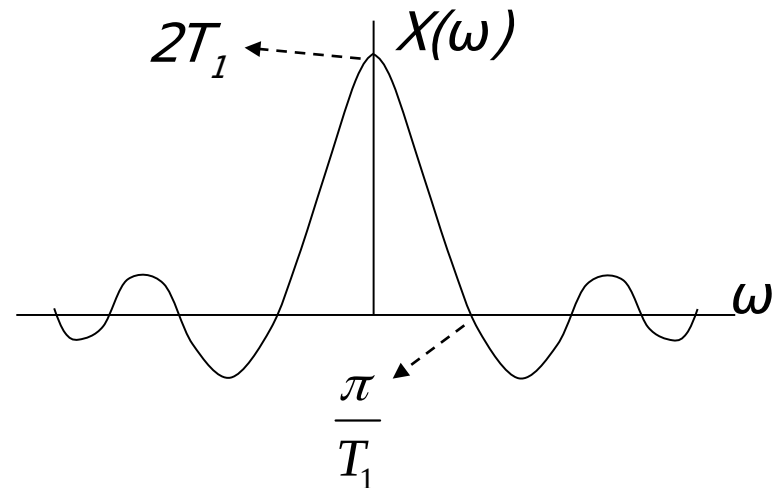
Example

a) $x(t) = \delta(t) \Rightarrow X(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1$

b) Rectangular pulse signal $x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases} \rightarrow$

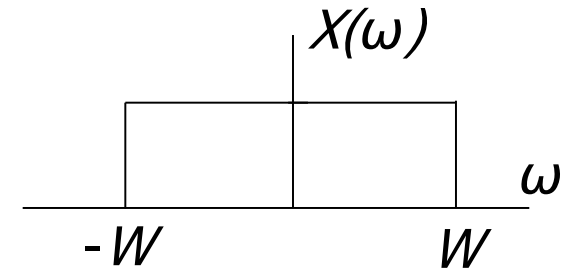


$$X(\omega) = \int_{-T_1}^{T_1} e^{-j\omega t} dt = \frac{2 \sin \omega T_1}{\omega} \rightarrow$$

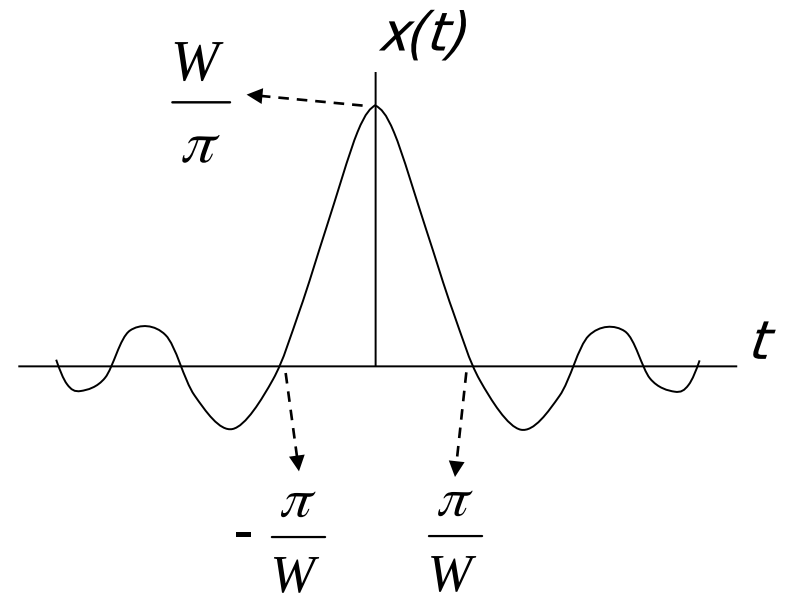


Example (cont'd)

c) Consider $x(t)$ with FT $X(\omega) = \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases} \rightarrow$



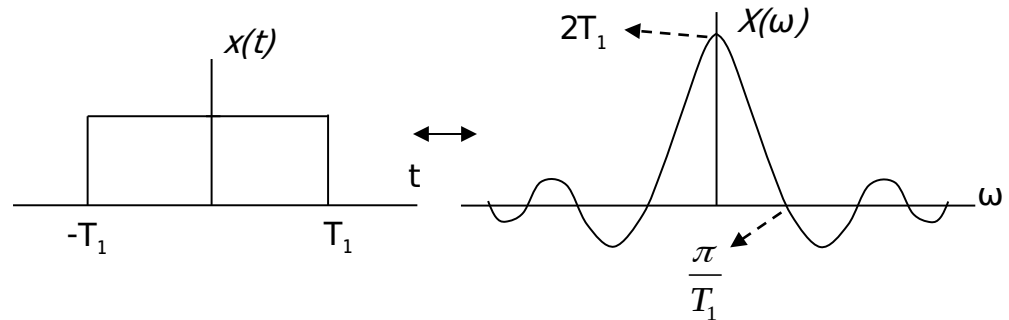
$$x(t) = \frac{1}{2\pi} \int_{-W}^W e^{j\omega t} d\omega = \frac{\sin Wt}{\pi t} \rightarrow$$



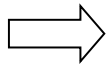
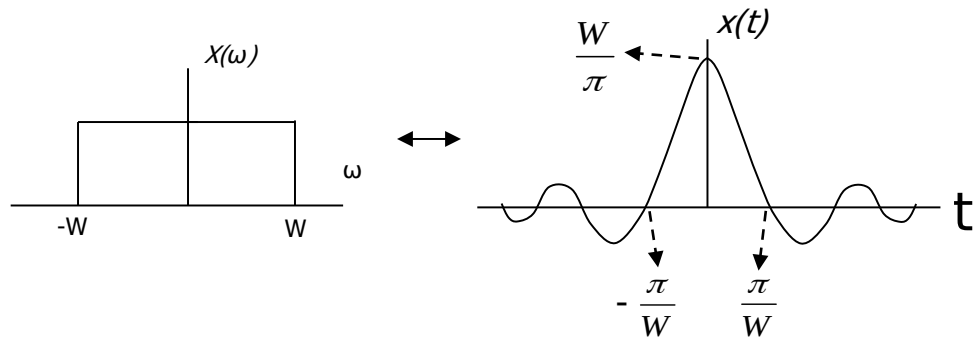
Note

$$\text{sinc}(\theta) = \frac{\sin \pi\theta}{\pi\theta} \quad \text{important function}$$

$$X(\omega) = \frac{2 \sin \omega T_1}{\omega} = 2T_1 \text{sinc}\left(\frac{\omega T_1}{\pi}\right)$$



$$x(t) = \frac{\sin Wt}{\pi t} = \frac{W}{\pi} \text{sinc}\left(\frac{Wt}{\pi}\right)$$



Narrow in Time Domain $\rightarrow$ have Broad FT!

Broad in Time Domain $\rightarrow$ Narrow FT!

(Scaling property)

Fourier Transform For Periodic Signals

Consider a signal $x(t)$ with Fourier Transform $X(\omega) = 2\pi \cdot \delta(\omega - \omega_0)$
(i.e., a signal impulse of area 2π at $\omega = \omega_0$)

Let us find the signal $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi \delta(\omega - \omega_0) e^{j\omega t} d\omega = e^{j\omega_0 t}$

if $X(\omega)$ is a linear combination of impulses equally spaced in frequency,
i.e.,

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

then $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$: *FS representation of periodic signal*

Fourier Transform For Periodic Signals

Hence, FT of periodic signal is weighted impulse train occurring at integer multiples of ω_0

Example:

a) Periodic square wave:

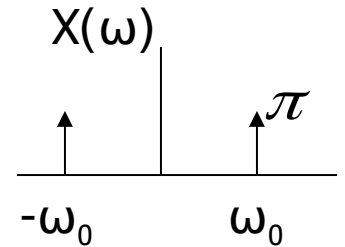
$$a_k = \frac{\sin k\omega_0 T_1}{\pi k} \quad \Rightarrow \quad X(\omega) = \sum_k \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$$

Recall FS coefficients of periodic square wave (from previous chapter)

Fourier Transform For Periodic Signals

$$\text{b) } x(t) = \sin \omega_0 t \longrightarrow a_k = \begin{cases} 1/2j, & k = 1 \\ -1/2j, & k = -1 \\ 0, & \text{o/w} \end{cases} \Rightarrow X(\omega) = j\pi \{ \delta(\omega + \omega_0) - \delta(\omega - \omega_0) \}$$

$$x(t) = \cos \omega_0 t \longrightarrow X(\omega) = \pi \{ \delta(\omega + \omega_0) + \delta(\omega - \omega_0) \}$$



$$\text{c) } x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) \quad \text{impulse train}$$

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} \delta(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \Rightarrow X(\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T} \delta\left(\omega - \frac{2\pi k}{T}\right)$$

FT is again impulse train in frequency domain with period $\frac{2\pi}{T}$

Properties of CT Fourier Transform

Notations: $x(t) \xleftarrow{F} X(\omega)$, or $X(\omega) \triangleq F\{x(t)\}$

1. Linearity:

If $x(t) \xleftarrow{F} X(\omega)$, $y(t) \xleftarrow{F} Y(\omega)$,

then $ax(t) + by(t) \xleftarrow{F} aX(\omega) + bY(\omega)$,

Prove it as an exercise!

Properties of CT Fourier Transform

2. Time Shifting:

If $x(t) \xleftrightarrow{F} X(\omega)$, then $x(t - t_0) \xleftrightarrow{F} e^{-j\omega t_0} X(\omega)$

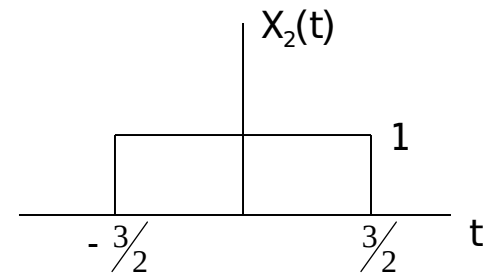
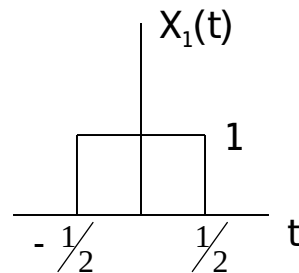
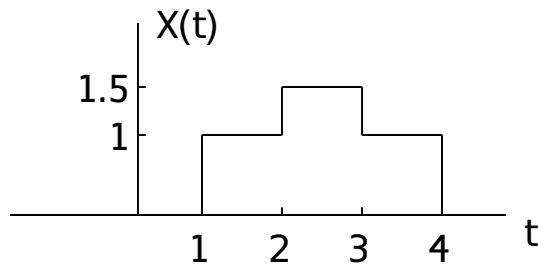
Proof:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

$$\begin{aligned} \rightarrow x(t - t_0) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega(t - t_0)} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \underbrace{(e^{-j\omega t_0} X(\omega))}_{F\{x(t-t_0)\}} e^{j\omega t} d\omega \end{aligned}$$

Properties of CT Fourier Transform

Example



$$x(t) = \frac{1}{2} x_1(t - 2.5) + x_2(t - 2.5)$$

$$X_1(\omega) = \frac{2 \sin(\omega/2)}{\omega}, \quad X_2(\omega) = \frac{2 \sin(3\omega/2)}{\omega}$$

$$\Rightarrow X(\omega) = e^{-j5\frac{\omega}{2}} \left\{ \frac{\sin(\omega/2) + 2 \sin(3\omega/2)}{\omega} \right\}$$

By linearity and
time-shifting properties
of FT

Properties of CT Fourier Transform

3. Conjugation / Conjugate Symmetry:

$$x(t) \xleftrightarrow{F} X(\omega) \Rightarrow x^*(t) \xleftrightarrow{F} X^*(-\omega)$$

for real $x(t)$ $X(-\omega) = X^*(\omega)$ *Opp. pg. 303 for proof!*

4. Differentiation & Integration:

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega X(\omega) e^{j\omega t} d\omega \Rightarrow \frac{dx(t)}{dt} \xleftrightarrow{F} j\omega X(\omega)$$

↗
Multiplication in
frequency domain

* Important in solving linear differential equations:

$$\int_{-\infty}^t x(\tau) d\tau \xleftrightarrow{F} \frac{1}{j\omega} X(\omega) + \pi X(0) \delta(\omega)$$

Properties of CT Fourier Transform

Example

Consider $\frac{dy(t)}{dt} + \alpha y(t) = x(t)$ Take FT of both sides;

$$(j\omega + \alpha)Y(\omega) = X(\omega) \longrightarrow Y(\omega) = \frac{1}{\alpha + j\omega} X(\omega)$$

$$\text{For } x(t) = \delta(t) \longrightarrow X(\omega) = 1 \implies H(\omega) = \frac{1}{\alpha + j\omega}$$

$$\implies h(t) = e^{-\alpha t} u(t)$$

Properties of CT Fourier Transform

Example

Consider $x(t) = u(t)$ (unit step)

For $g(t) = \delta(t)$, $u(t) = \int_{-\infty}^t g(\tau) d\tau$

$$\Rightarrow u(t) \xleftarrow{F} \frac{1}{j\omega} G(\omega) + \pi G(0) \delta(\omega)$$

$$\Rightarrow u(t) \xleftarrow{F} \frac{1}{j\omega} + \pi \delta(\omega)$$

Properties of CT Fourier Transform

5. Time and Frequency Scaling:

$$\text{If } x(t) \xleftrightarrow{F} X(\omega), \quad \text{then} \quad x(at) \xleftrightarrow{F} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

(Prove using FT integral)

Remark:

- Inverse relation between time and frequency domains:
- A signal varying rapidly will have a transform occupying wider frequency band, and vice versa
- $x(-t) \xleftrightarrow{F} X(-\omega)$

Properties of CT Fourier Transform

6. Duality:

Observe the FT and inverse FT integrals:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt ; \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Recall

*Symmetry
between
the FT
pairs!*

$$x_1(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases} \xleftrightarrow{F} X_1(\omega) = \frac{2 \sin \omega T_1}{\omega}$$

$$x_2(t) = \frac{\sin Wt}{\pi t} \xleftrightarrow{F} X_2(\omega) = \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases}$$

In general;

$\text{if } g(t) \xleftrightarrow{F} f(\omega), \quad \text{then } f(t) \xleftrightarrow{F} 2\pi g(-\omega)$
--

Properties of CT Fourier Transform

Example:

$$x(t) = \delta(t) \quad \xleftrightarrow{F} \quad X(\omega) = 1$$

$$\text{By duality; } x(t) = 1 \quad \xleftrightarrow{F} \quad 2\pi \delta(\omega) \quad (\text{prove it!})$$

Dual of the properties:

$$- \quad jtx(t) \quad \xleftrightarrow{F} \quad \frac{dX(\omega)}{d\omega} \quad (\text{frequency differentiation})$$

$$e^{j\omega_0 t} x(t) \quad \xleftrightarrow{F} \quad X(\omega - \omega_0) \quad (\text{frequency shifting})$$

$$- \quad \frac{1}{jt} x(t) + \pi x(0)\delta(t) \quad \xleftrightarrow{F} \quad \int_{-\infty}^{\omega} X(\eta) d\eta$$

Properties of CT Fourier Transform

7. Parseval's Relation:

$$\text{if } x(t) \xleftrightarrow{F} X(\omega) \text{ then } \underbrace{\int_{-\infty}^{\infty} |x(t)|^2 dt}_{\text{total energy in } x(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

(Check the proof in Opp. pg.312)

total energy in $x(t)$

$|X(\omega)|^2$: energy density spectrum

Properties of CT Fourier Transform

Convolution Property

(Additional & very important properties of FT, in terms of LTI systems)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

Take FT. $\longrightarrow$

$$\begin{aligned} Y(\omega) &= \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau \right\} e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} x(\tau) \left\{ \int_{-\infty}^{\infty} h(t - \tau)e^{-j\omega t} dt \right\} d\tau \\ &= \int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau} H(\omega)d\tau = H(\omega) \int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau} d\tau \end{aligned}$$

$$Y(\omega) = H(\omega)X(\omega)$$

Properties of CT Fourier Transform

Convolution Property

$$h(t) \star x(t) \xleftrightarrow{F} H(\omega) X(\omega)$$

$\Rightarrow$ *Convolution of two signals in time domain is equivalent to multiplication of their spectrums in frequency domain*

$\Rightarrow$ $H(\omega)$: frequency response of the system

Example:

a) Consider a CT, LTI system with $h(t) = \delta(t - t_0)$

$\rightarrow$ Frequency response is $H(\omega) = e^{-j\omega t_0}$ *Recall the time-shifting property of FT*

$\Rightarrow Y(\omega) = H(\omega) X(\omega) = e^{-j\omega t_0} X(\omega) \rightarrow y(t) = x(t - t_0)$

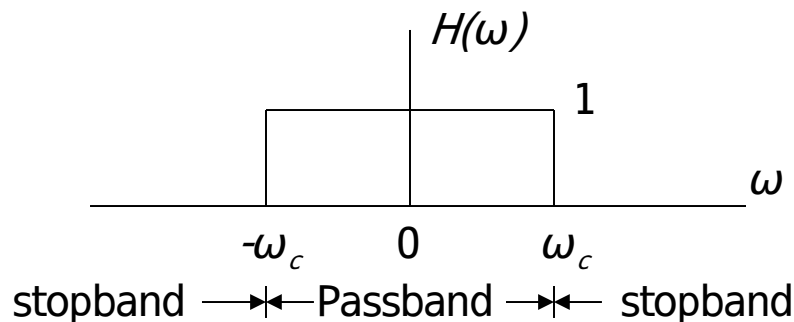
Properties of CT Fourier Transform

Convolution Property

b) *Frequency-selective filtering:*

achieved by an LTI system whose frequency response $H(\omega)$ passes desired range of frequencies and stops (attenuates) other frequencies, e.g.,

$$H(\omega) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases} ; \quad h(t) = \frac{\sin \omega_c t}{\pi t}$$



ideal lowpass filter

Properties of CT Fourier Transform

Multiplication Property

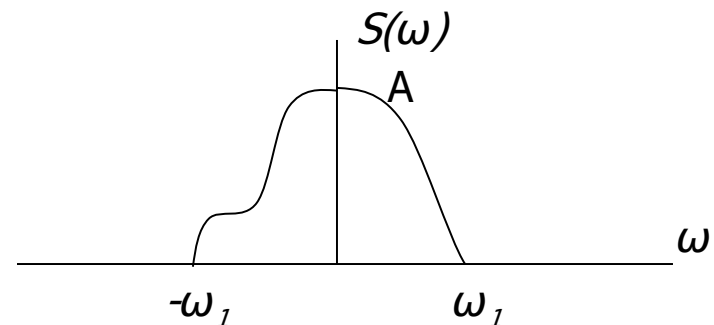
From convolution property and duality, multiplication in time domain corresponds to convolution in frequency domain.

$$r(t) = s(t)p(t) \quad \xleftrightarrow{F} \quad R(\omega) = \frac{1}{2\pi} [S(\omega) \star P(\omega)]$$

→ Amplitude Modulation (multiplication of two signals)
important in telecommunications !

Example:

Let $s(t)$ has spectrum $S(\omega)$



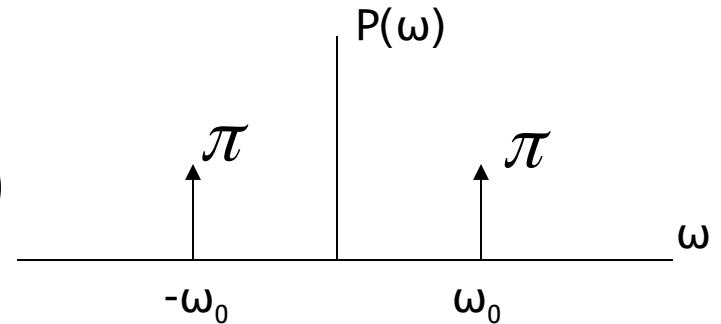
Baseband signal

Properties of CT Fourier Transform

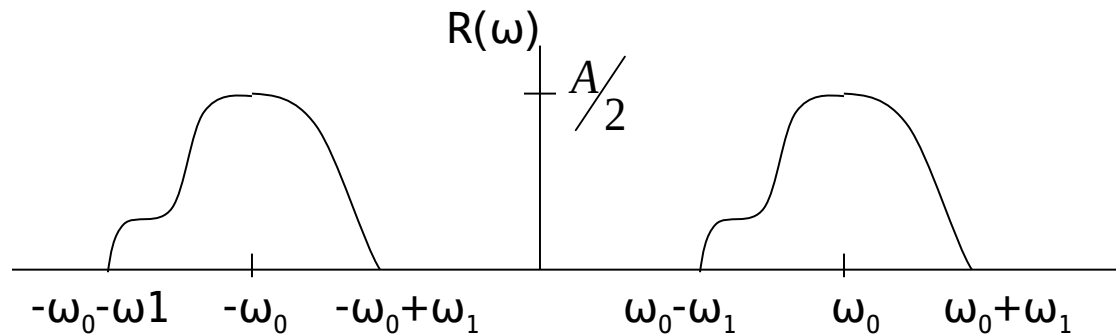
Multiplication Property

$$p(t) = \cos \omega_0 t$$

$$\Rightarrow P(\omega) = \pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)$$



$$R(\omega) = \frac{1}{2\pi} (S(\omega) \star P(\omega))$$



- Information (spectral content) in $s(t)$ is preserved but shifted to higher frequencies (more suitable for transmission)

Properties of CT Fourier Transform

Multiplication Property

To recover:

- Multiply $r(t)$ with $p(t)=\cos\omega_0 t \implies g(t)=r(t).p(t)$
- Apply lowpass filter!

Application of Fourier Theory

Communication Systems

Definitions:

- **Modulation:** Embedding an info-bearing signal into a second signal.
- **Demodulation:** Extracting the information bearing signal from the second signal.
- **Info-bearing signal $x(t)$:** The signal to be transmitted (modulating signal).
- **Carrier signal $c(t)$:** The signal which carries the info-bearing signal (usually a sinusoidal signal).

Application of Fourier Theory

Communication Systems
Modulated signal $y(t)$ is then the product of $x(t)$ and $c(t)$

$$y(t) = x(t) \cdot c(t)$$

Objective: To produce a signal whose frequency range is suitable for transmission over communication channel. e.g.:

- individual voice signals are in 200Hz-4kHz
- telephony (long-distance) over microwave or satellite links in 300MHz-300GHz (microwave), 300MHz-40GHz (satellite)

⇒ Information in voice signals must be shifted into these higher ranges of frequency

Application of Fourier Theory

Amplitude Modulation with Complex Exponential Carrier

$$c(t) = e^{j(\omega_c t + \theta_c)} \quad \omega_c: \text{carrier frequency}$$

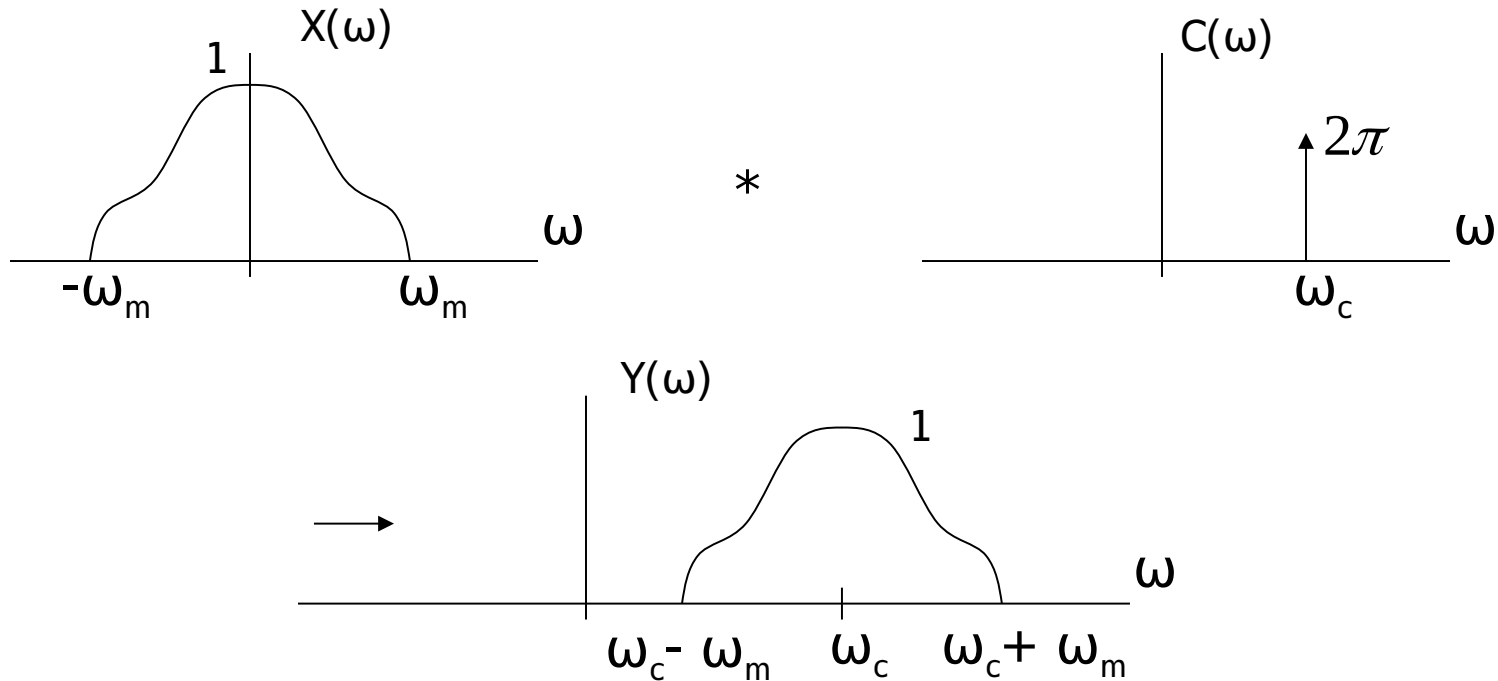
Consider $\theta_c = 0 \quad \Rightarrow \quad y(t) = x(t)e^{j\omega_c t}$

From multiplication property $\rightarrow Y(\omega) = \frac{1}{2\pi} X(\omega) \star C(\omega)$

$$C(\omega) = 2\pi \delta(\omega - \omega_c) \rightarrow Y(\omega) = X(\omega - \omega_c)$$

Application of Fourier Theory

Amplitude Modulation with Complex Exponential Carrier



To recover $x(t)$ from $y(t)$: $x(t) = y(t)e^{-j\omega_c t}$ (Shift the spectrum back)

(Demodulation)

Application of Fourier

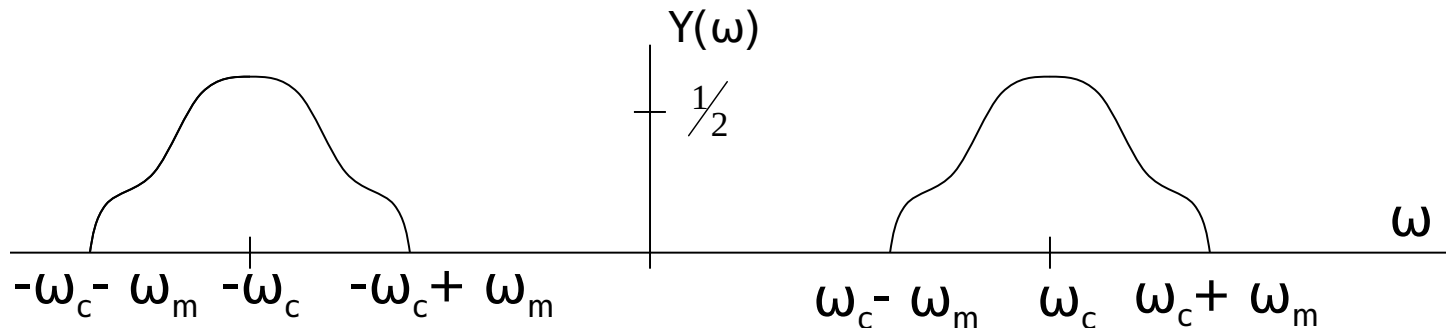
Theory *Amplitude Modulation with Sinusoidal Carrier*

$$c(t) = \cos(\omega_c t + \theta_c)$$

$$\begin{array}{ccccc}
 & c(t) = \cos(\omega_c t + \theta_c) & & & \\
 & \downarrow & & & \\
 x(t) & \longrightarrow & \otimes & \longrightarrow & y(t)
 \end{array}$$

For $\theta_c = 0$, $y(t) = x(t) \cos \omega_c t$, $C(\omega) = \pi [\delta(\omega + \omega_c) + \delta(\omega - \omega_c)]$

$$Y(\omega) = \frac{1}{2} [X(\omega + \omega_c) + X(\omega - \omega_c)]$$

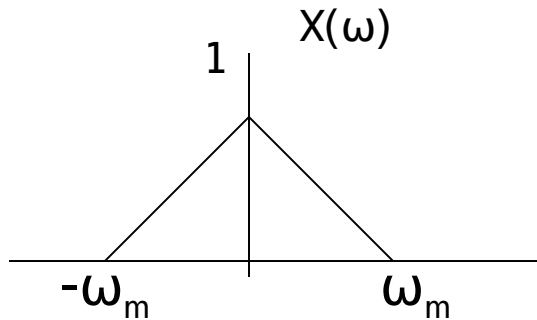


Application of Fourier

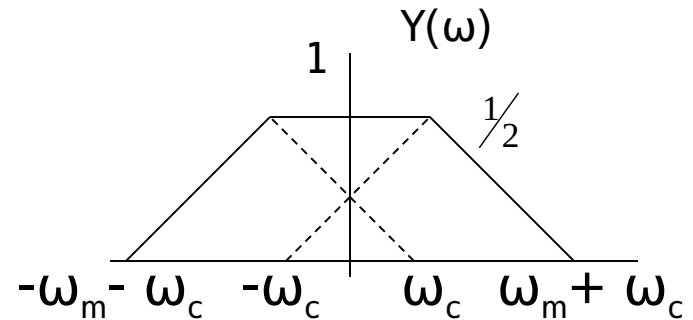
Theory *Amplitude Modulation with a Sinusoidal Carrier*

To recover $x(t)$ from $y(t)$, the condition $\omega_c > \omega_m$ must be satisfied! Otherwise, the replicas will overlap.

Example:



$$\text{For } \omega_c = \frac{\omega_m}{2}$$



Application of Fourier Theory *Demodulation for Sinusoidal Amplitude Modulation*

→ To recover the information-bearing signal $x(t)$ at the receiver

Synchronous Demodulation: Transmitter and receiver are synchronized in phase

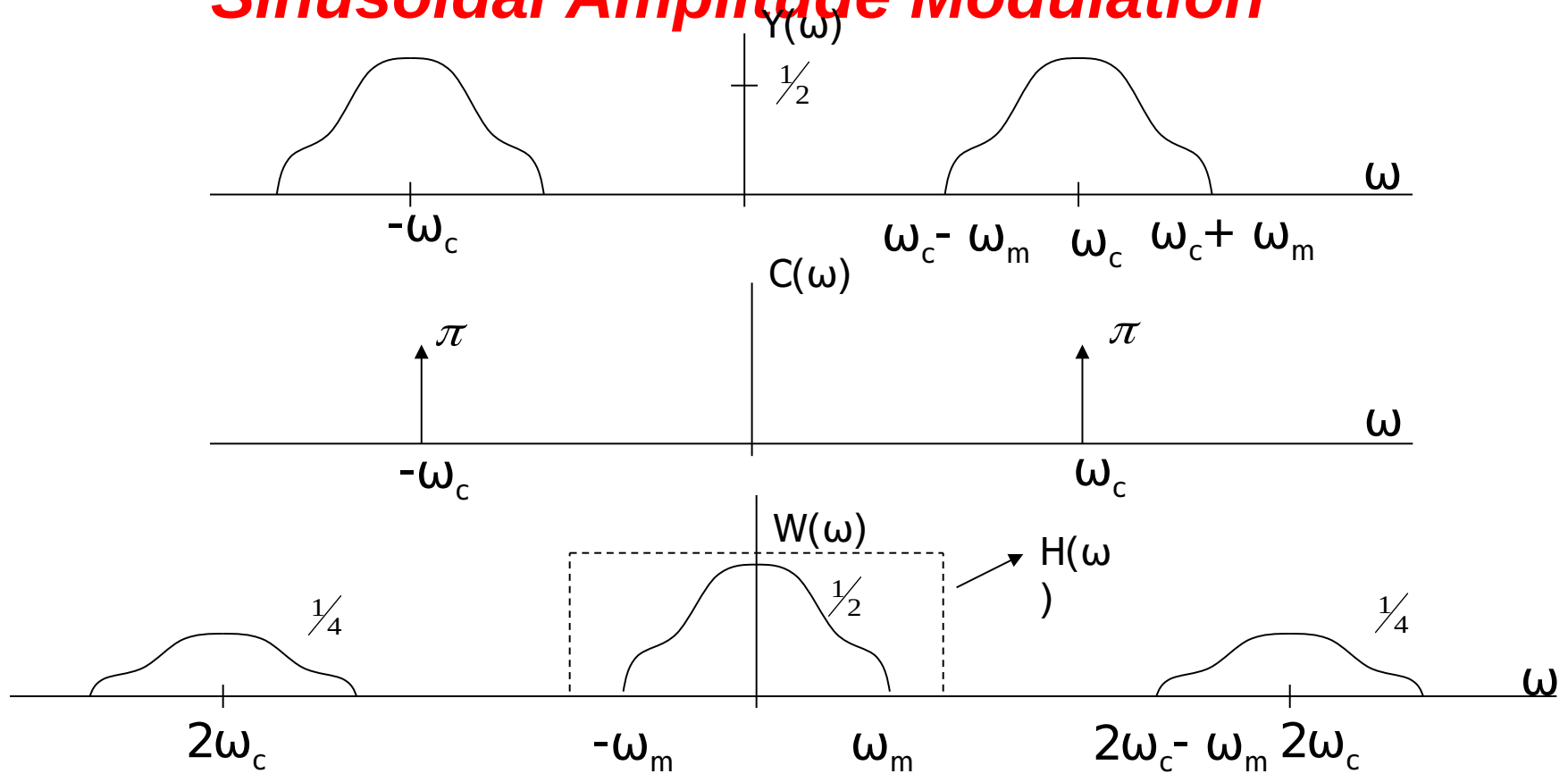
$y(t) = x(t) \cos \omega_c t$: $x(t)$ can be recovered by modulating $y(t)$ with the same sinusoidal carrier and applying a lowpass filter.

$$\begin{aligned} w(t) &= y(t) \cos \omega_c t = x(t) \cos^2 \omega_c t \\ &= x(t) \left(\frac{1}{2} + \frac{1}{2} \cos 2\omega_c t \right) \\ w(t) &= \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos 2\omega_c t \end{aligned}$$

(need to get rid of the 2nd term in RHS)

Application of Fourier Theory

Demodulation for Sinusoidal Amplitude Modulation

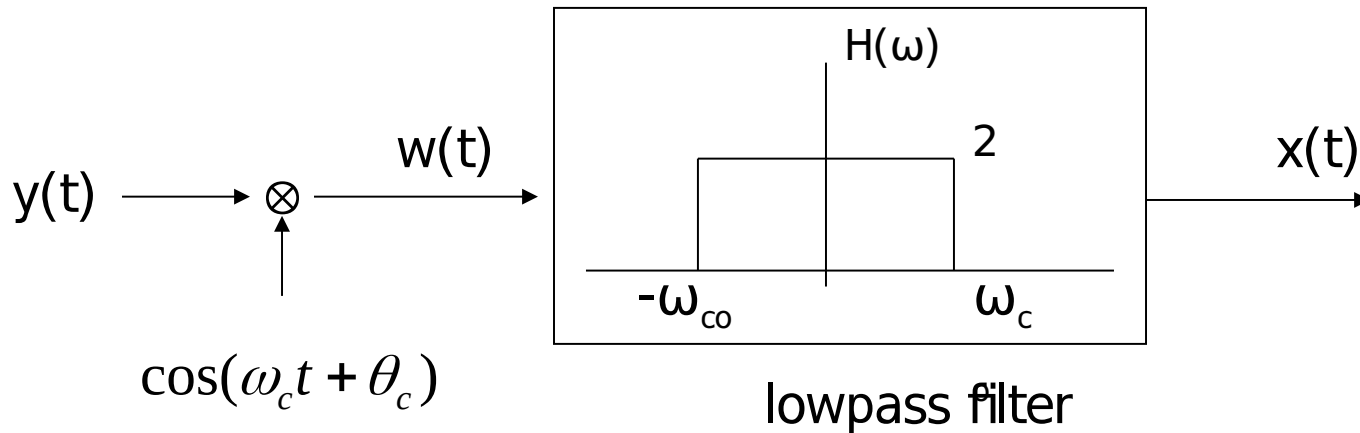
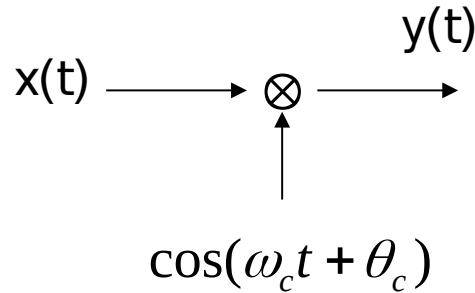


Apply lowpass filter ($H(\omega)$) with a gain of 2 and cutoff frequency (ω_{co})

$$\omega_m < \omega_{co} < 2\omega_c - \omega_m$$

Application of Fourier Theory *Demodulation for Sinusoidal Amplitude Modulation*

In general,



Application of Fourier

Theory *Demodulation for Sinusoidal Amplitude Modulation*

Assume that modulator and demodulator are not synchronized;

θ_c : phase of modulator

φ_c : phase of demodulator

$$\Rightarrow w(t) = x(t) \cos(\omega_c t + \theta_c) \cos(\omega_c t + \varphi_c)$$

$$= x(t) \left\{ \frac{1}{2} \cos(\theta_c - \varphi_c) + \frac{1}{2} \cos(2\omega_c t + \theta_c + \varphi_c) \right\}$$

$$w(t) = \frac{1}{2} \cos(\theta_c - \varphi_c) x(t) + \frac{1}{2} x(t) \cos(2\omega_c t + \theta_c + \varphi_c)$$

Application of Fourier Theory *Demodulation for Sinusoidal Amplitude Modulation*

When we apply lowpass filter $\longrightarrow \cos(\theta_c - \varphi_c)x(t)$

if $\theta_c = \varphi_c \longrightarrow \text{output } x(t)$

$\theta_c - \varphi_c = \frac{\pi}{2} \longrightarrow \text{output } 0$

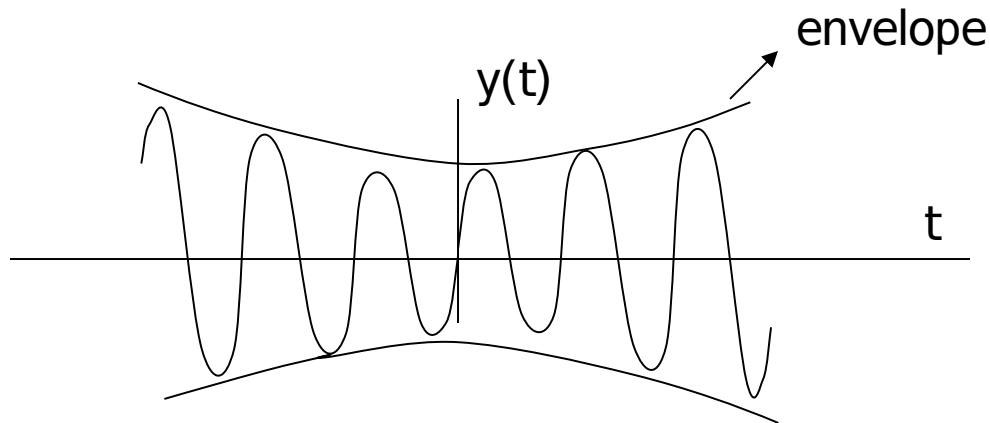
$(\theta_c - \varphi_c)$ must be maintained over time $\longrightarrow$ requires synchronization

Application of Fourier

Theory *Demodulation for Sinusoidal Amplitude Modulation*

Asynchronous Demodulation:

- Avoids the need for synchronization between the modulator and demodulator
- If the message signal $x(t)$ is positive, and carrier frequency ω_c is much higher than ω_m (the highest frequency in the modulating signal), then envelope of $y(t)$ is a very close approximation to $x(t)$



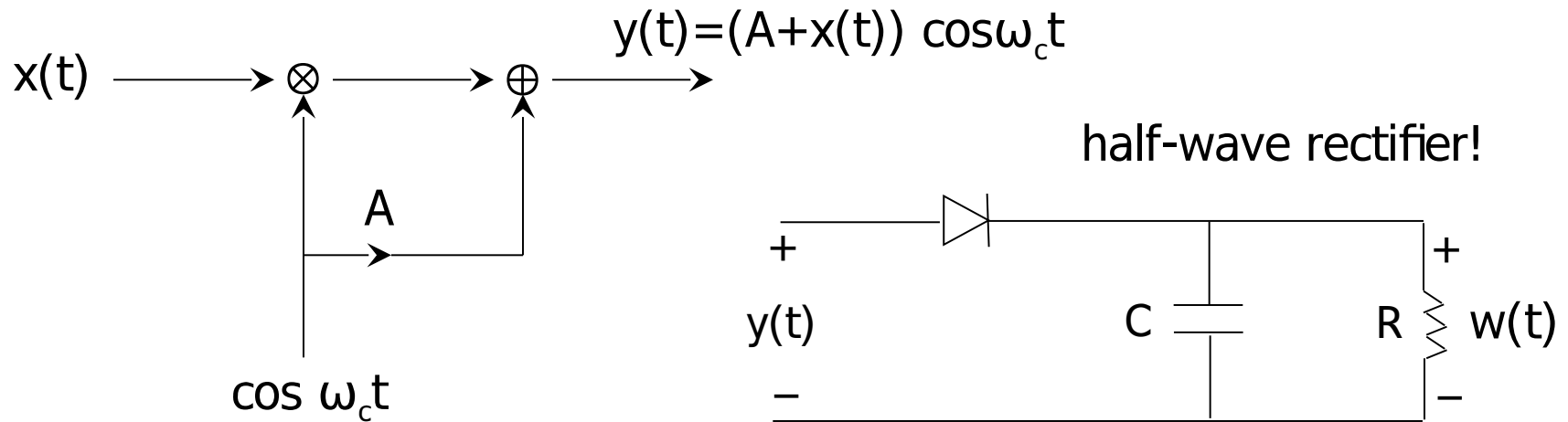
Application of Fourier

Theory *Demodulation for Sinusoidal Amplitude Modulation*

⇒ Envelope Detector :

→ to assure positivity add DC to message signal, i.e., $x(t) + A > 0$

→ $x(t)$ vary slowly compared to ω_c (to track envelope)

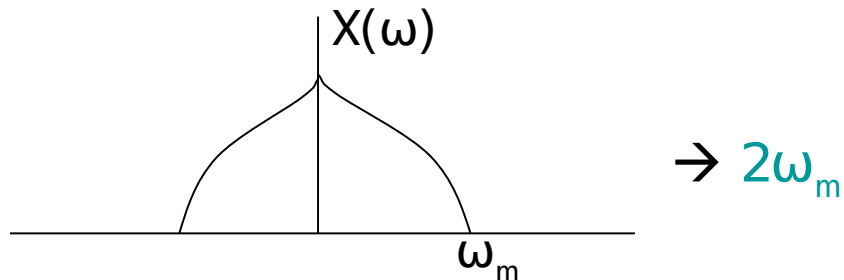


⇒ Tradeoff : simpler demodulator, but requires transmission of redundancy (higher power)

Application of Fourier Theory

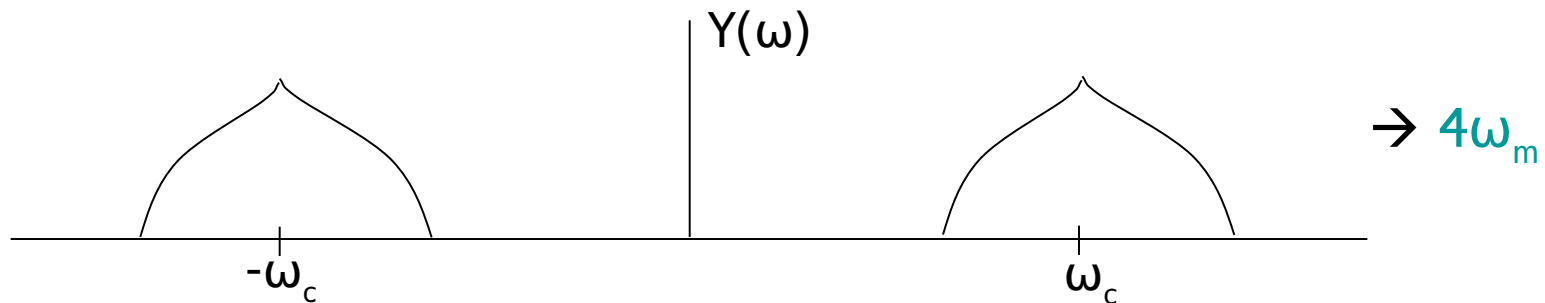
Single-Sideband (SSB) Sinusoidal Amplitude Modulation

For ω_m is the highest frequency in $x(t)$, total bandwidth of the original signal $2\omega_m$.



With sinusoidal carrier: spectrum shifted to ω_c and $-\omega_c$

→ twice bandwidth is required.

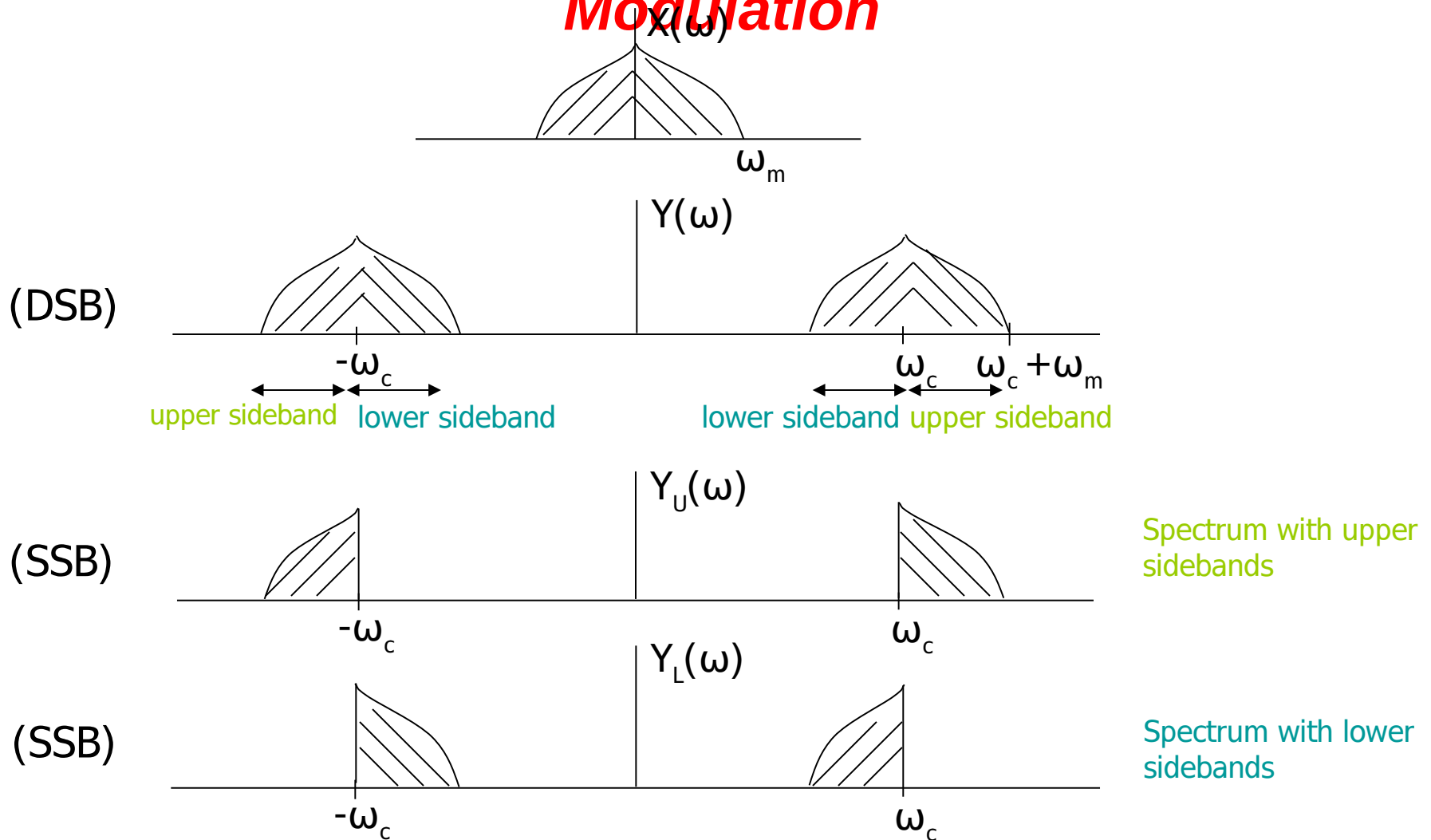


→ Redundancy in modulated signal!

→ *Solution: Use SSB modulation*

Application of Fourier Theory

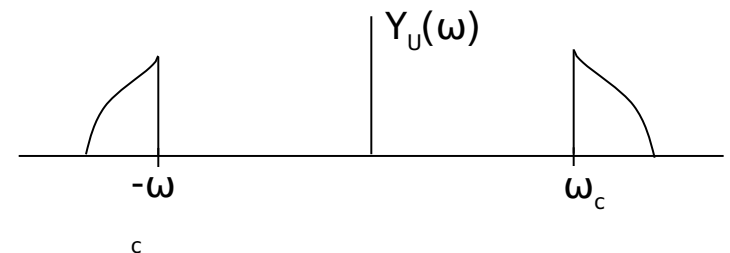
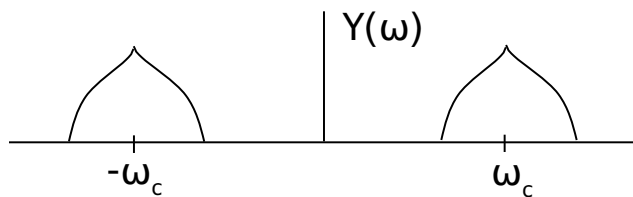
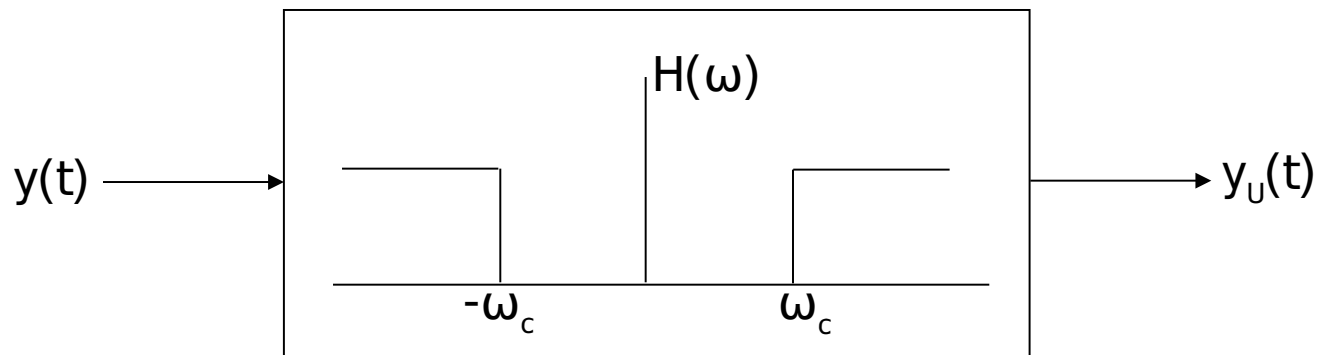
Single-Sideband (SSB) Sinusoidal Amplitude Modulation



Application of Fourier Theory

Single-Sideband (SSB) Sinusoidal Amplitude Modulation

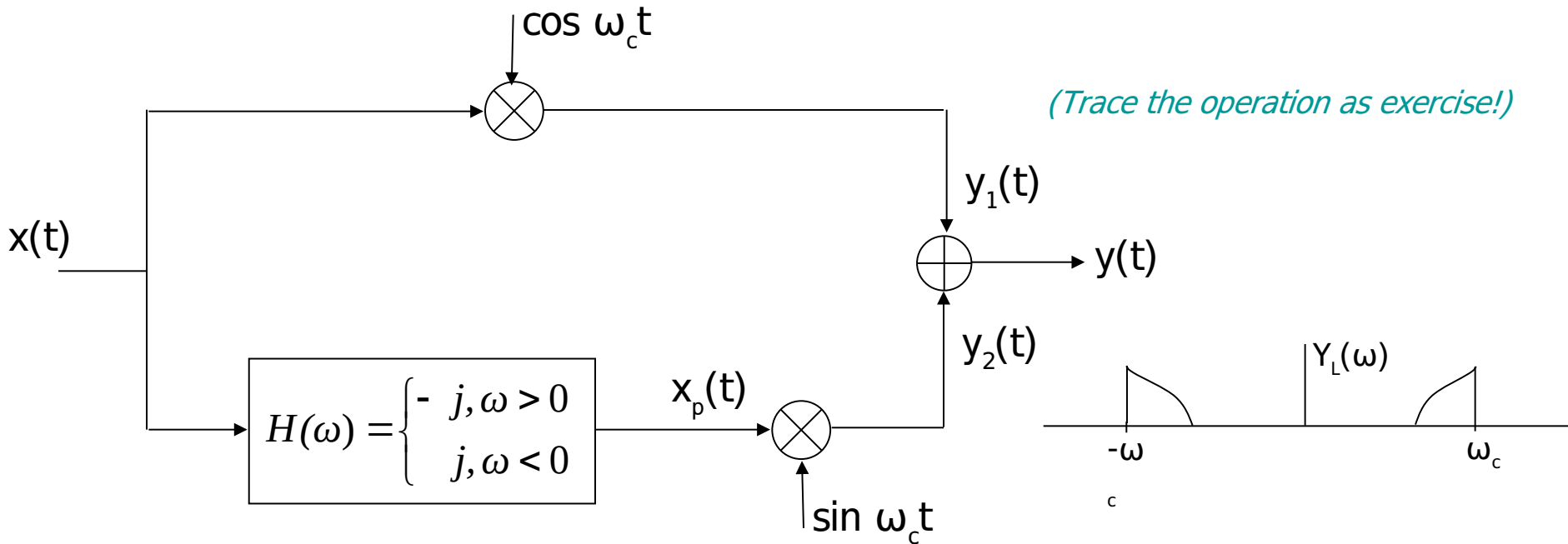
For upper sidebands: Apply $y(t)$ to a sharp cutoff bandpass/highpass filter.



Application of Fourier Theory

Single-Sideband (SSB) Sinusoidal Amplitude Modulation

For lower sidebands: Use 90° phase-shift network.



AM-DSB/WC , AM-SSB/WC : AM, Double (Single) SB, with carrier

AM-DSB/SC , AM-SSB/SC : AM, Double (Single) SB, suppressed carrier