

FOURIER DÖNÜŞÜMÜ

$$\bullet F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} \cdot dt$$

$$\bullet F(f) = F(f(t)) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j2\pi f t} \cdot dt$$

$$\bullet f(t) = F^{-1}(F(\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \cdot e^{j\omega t} \cdot d\omega$$

$$\bullet f(f) = F^{-1}(F(f)) = \int_{-\infty}^{\infty} F(f) \cdot e^{j2\pi f t} \cdot df$$



* Bir işaretin Fourier dönüşümünün alınabilmesi için;

$$\int_{-\infty}^{\infty} |f(t)| dt < \infty \text{ şartı sağlanmalıdır.}$$

* Fourier dönüşümü ile Fourier serisi katsayıları arasındaki ilişki;

$$F_k = \frac{1}{T_0} F(\omega) \Big|_{\omega = k\omega_0} \Rightarrow F_k = \frac{1}{T_0} F(k\omega_0)$$

* Periyodik olmayan işaretlerin genlik ve faz tayfi süreklidir.

* Periyodik işaretlerin Fourier dönüşümü hesaplanamaz, fakat dolaylı olarak hesaplanabilir. Bu işaretlerin genlik ve faz tayfi çizgiden oluşur.

$$* x(t) = e^{+j\omega_0 t}$$

$$X(\omega) = 2\pi \delta(\omega - \omega_0)$$

$$X(f) = \delta(f - f_0)$$

$$* x(t) = e^{+jk\omega_0 t}$$

$$X(\omega) = 2\pi \delta(\omega - k\omega_0)$$

$$X(f) = \delta(f - kf_0)$$

$$* x(t) = e^{+j(k\omega_0 t + \phi)}$$

$$X(\omega) = e^{+j\phi} 2\pi \delta(\omega - k\omega_0)$$

$$X(f) = e^{+j\phi} \delta(f - kf_0)$$

$$* \text{Örneğin } x(t) = e^{j2\pi t}$$

$$\Rightarrow X(\omega) = 2\pi \delta(\omega - 2\pi)$$

$$X(f) = \delta(f - 1)$$

$$* \text{Örneğin } x(t) = \cos \omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$\Rightarrow X(\omega) = \frac{1}{2} (2\pi \delta(\omega - \omega_0) + 2\pi \delta(\omega + \omega_0)) \Rightarrow X(\omega) = \pi (\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$$

$$\Rightarrow X(f) = \delta(f - f_0) + \delta(f + f_0)$$

$$\Rightarrow X(f) = \frac{1}{2} (\delta(f - f_0) + \delta(f + f_0))$$

* Örneğin; $x(t) = \sin \omega_0 t = (e^{j\omega_0 t} - e^{-j\omega_0 t}) / 2j$

$\Rightarrow x(\omega) = \frac{1}{2j} (2\pi \delta(\omega - \omega_0) - 2\pi \delta(\omega + \omega_0))$

$\Rightarrow x(\omega) = \frac{\pi}{j} (\delta(\omega - \omega_0) - \delta(\omega + \omega_0))$

$\Rightarrow x(f) = \frac{1}{2j} (\delta(f - f_0) - \delta(f + f_0))$

pure $f''(f\omega)$

* Örneğin; $x(t) = e^{-j(2.5\pi t + \pi/4)}$

$\Rightarrow x(\omega) = e^{-j\pi/4} \cdot 2\pi \delta(\omega + 2.5\pi/2)$

$\Rightarrow x(f) = e^{-j\pi/4} \delta(f + 2.5/4)$

* Örneğin; $x(t) = \cos(\frac{2\pi}{3}t - \pi/3) = \frac{e^{j(\frac{2\pi}{3}t - \pi/3)} + e^{-j(\frac{2\pi}{3}t - \pi/3)}}{2}$

$\Rightarrow x(\omega) = \frac{1}{2} (e^{-j\pi/3} \cdot 2\pi \delta(\omega - 2\pi/3) + e^{+j\pi/3} \cdot 2\pi \delta(\omega + 2\pi/3))$

$\Rightarrow x(f) = \frac{1}{2} e^{-j\pi/3} \delta(f - 1/3) + \frac{1}{2} e^{+j\pi/3} \delta(f + 1/3)$

* Fourier Dönüşümünün Özellikleri

1) Doğrusallık özelliği

$f(t) \leftrightarrow F(\omega)$

$g(t) \leftrightarrow G(\omega)$

$a f(t) + b g(t) \leftrightarrow a F(\omega) + b G(\omega)$

2) Dikgenlik Özelliği

$\int_{-\infty}^{\infty} f(t) \cdot g^*(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \cdot G^*(\omega) \cdot d\omega = \int_{-\infty}^{\infty} F(f) \cdot G^*(f) \cdot df$

$f(t) = g(t) \Rightarrow \int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega = \int_{-\infty}^{\infty} |F(f)|^2 df$

Zaman domeninde

hesaplanan enerji

Frekans domeninde

hesaplanan enerji

PARSEVAL TEOREMİ!

3) Kayma Özellikleri

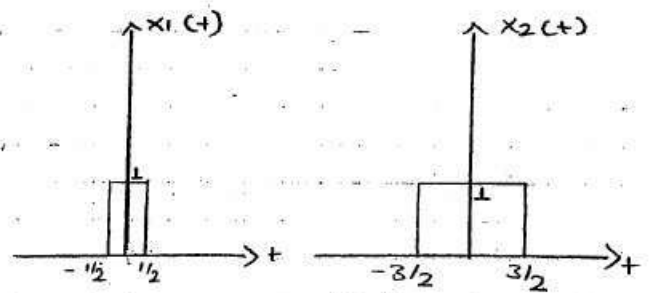
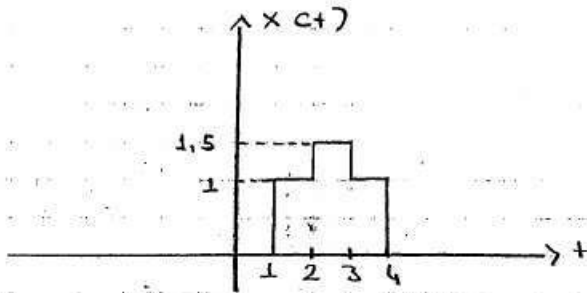
3-a) Zamanda Kayma Özelliği

$$f(t) \leftrightarrow F(\omega)$$

$$f(t-t_0) \leftrightarrow e^{-j\omega t_0} F(\omega)$$

t_0 : Kayma miktarı!

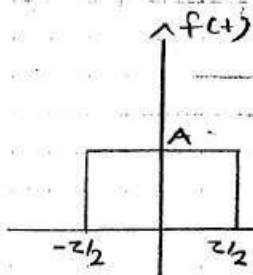
Örneği:



$x(t)$ 'yi Fourier dönüşümünü kolayca hesaplayabileceğimize

$x_1(t)$ ve $x_2(t)$ cinsinden ifade edebiliriz.

$$x(t) = \frac{1}{2} x_1(t-2.5) + x_2(t-2.5)$$



$$F(\omega) = A z \text{sinc}(fz) \Rightarrow$$

$$\begin{matrix} * x_1(t) \text{ için} \\ z=1 \end{matrix} \left. \begin{matrix} A=1 \\ \end{matrix} \right\} x_1(\omega) = \text{sinc}(f)$$

$$\begin{matrix} * x_2(t) \text{ için} \\ z=3 \end{matrix} \left. \begin{matrix} A=1 \\ \end{matrix} \right\} x_2(\omega) = 3 \text{sinc}(3f)$$

$$x_1(f) = \text{sinc}(f) \Rightarrow x_1(\omega) = \frac{\sin(\omega/2)}{\omega/2}$$

$$x_2(f) = 3 \text{sinc}(3f) \Rightarrow x_2(\omega) = \frac{3 \sin(3\omega/2)}{3\omega/2}$$

$$\Rightarrow X(\omega) = \frac{1}{2} e^{-j2.5\omega} \cdot \frac{\sin(\omega/2)}{\omega/2} + e^{-j2.5\omega} \cdot \frac{3 \sin(3\omega/2)}{3\omega/2}$$

$$\Rightarrow X(\omega) = e^{-j2.5\omega} \left(\frac{\sin(\omega/2)}{\omega} + \frac{2 \sin(3\omega/2)}{\omega} \right)$$

* Şimdi de formül den yazalım;

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt \Rightarrow X(\omega) = \int_1^2 e^{-j\omega t} dt + \frac{3}{2} \int_2^4 e^{-j\omega t} dt + \int_3^4 e^{-j\omega t} dt$$

$$\Rightarrow x(\omega) = \frac{-1}{j\omega} e^{-j\omega \frac{2}{1}} - \frac{3}{2j\omega} e^{-j\omega \frac{3}{2}} - \frac{1}{j\omega} e^{-j\omega \frac{4}{3}}$$

$$\Rightarrow x(\omega) = \frac{-1}{j\omega} (e^{-j2\omega} - e^{-j\omega}) - \frac{3}{2j\omega} (e^{-j3\omega} - e^{-j2\omega}) - \frac{1}{j\omega} (e^{-j4\omega} - e^{-j3\omega})$$

$$\Rightarrow x(\omega) = \frac{-1}{j\omega} e^{-j2\omega} + \frac{1}{j\omega} e^{-j\omega} - \frac{3}{2j\omega} e^{-j3\omega} + \frac{3}{2j\omega} e^{-j2\omega} - \frac{1}{j\omega} e^{-j4\omega} + \frac{1}{j\omega} e^{-j3\omega}$$

$$\Rightarrow x(\omega) = \frac{1}{j2\omega} e^{-j2\omega} + \frac{1}{j\omega} e^{-j\omega} - \frac{1}{2j\omega} e^{-j3\omega} - \frac{1}{j\omega} e^{-j4\omega}$$

$$x(\omega) = \frac{e^{-j2.5\omega}}{j2\omega} (e^{j\omega/2} - e^{-j\omega/2}) + \frac{1}{j\omega} e^{-j2.5\omega} (e^{j3\omega/2} - e^{-j3\omega/2})$$

$$\Rightarrow x(\omega) = e^{-j2.5\omega} \times \sin(\omega/2)/\omega + 2e^{-j2.5\omega} \times \sin(3\omega/2)/\omega$$

$$\Rightarrow x(\omega) = e^{-j2.5\omega} (\sin(\omega/2)/\omega + 2\sin(3\omega/2)/\omega)$$

Aynı sonucu verdi!

3-b) Zamanda Katlama Özelliği

Sözü ifadesi: Bir işaret herhangi bir işaretin katlanmasıyla meydana geliyorsa bu işaretin fourier dönüşümü, diğer bir işaretin fourier dönüşümünün çarpımıdır.

$$x(t) = f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau) g(t-\tau) d\tau$$

$$\Rightarrow x(\omega) = F(\omega) \cdot G(\omega)$$

Example: $h(t) = \delta(t-t_0)$, $x(t)$ sürekli bir sinyal ve fourier dönüşümü $F\left\{\int_{-\infty}^{\infty} x(t) dt\right\} = X(\omega)$ dir.

$$y(t) = x(t) * h(t) = x(t-t_0)$$

Çarpımı $x(t_0)$, katlaması $x(t-t_0)$

$$\Rightarrow Y(\omega) = X(\omega) \cdot H(\omega) = X(\omega) \cdot e^{-j\omega t_0}$$

$\delta(t-t_0)$ 'in fourier dönüşümü!

$$* |e^{-j\omega t_0}| = 1, \quad -\omega t_0 \text{ faz özelliği!}$$

3-c) Modülasyon Özelliği

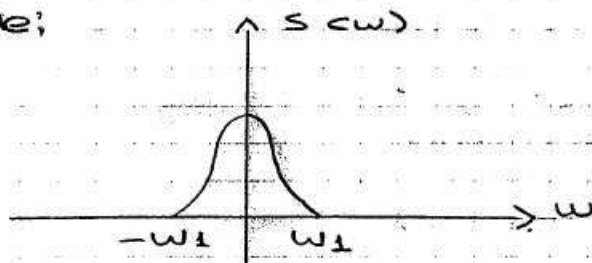
$$e^{j\omega_0 t} \cdot f(t) \leftrightarrow F(\omega - \omega_0)$$

3-d) Zamanda Garpma Özelliği

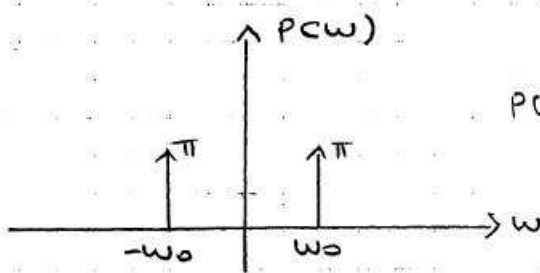
Sözlü ifadesi: Bir işaret herhangi ikt işaretin garpımı ise bu işaretin fourier dönüşümü diğer ikt işaretin fourier dönüşümünün tethamasının 2π 'ye bölümüdür.

$$x(t) = f(t) \cdot g(t) \Rightarrow X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega - u) \cdot G(u) \, du = \frac{1}{2\pi} F(\omega) * G(\omega)$$

Example;



Şekil: $s(t)$ işaretinin fourier dönüşümü



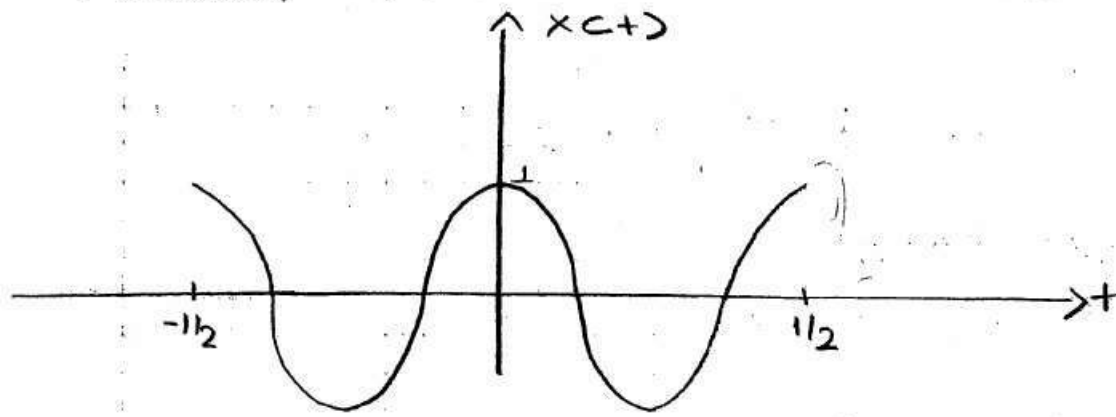
$$p(\omega) = \pi (\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$$

Şekil: $r(t) = \cos \omega_0 t$ nin fourier dönüşümü

$$r(t) = p(t) \cdot s(t) \Rightarrow R(\omega) = \frac{1}{2\pi} p(\omega) * s(\omega)$$

$$\Rightarrow R(\omega) = \frac{1}{2} (s(\omega - \omega_0) + s(\omega + \omega_0))$$

ÖRNEK:



*2amanda carpma özelliğinden yararlanarak $x(t)$ 'yi bulalım.

$$x(t) = \underbrace{\cos t}_{f(t)} + \underbrace{(u(t+1/2) - u(t-1/2))}_{g(t)}$$

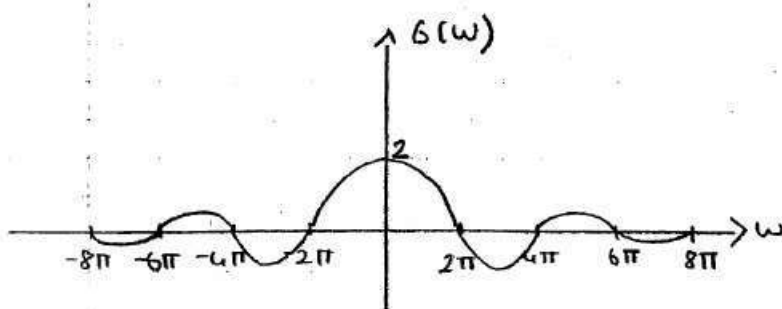
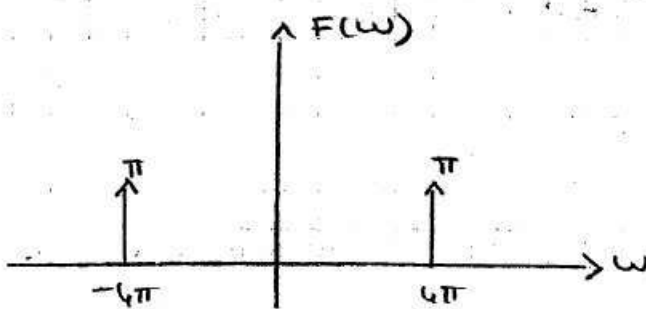
$$f(t) = \cos t \Rightarrow F(\omega) = (e^{j\omega t} + e^{-j\omega t})/2$$

$$\Rightarrow F(\omega) = \frac{1}{2} (2\pi \delta(\omega - 1) + 2\pi \delta(\omega + 1))$$

$$\Rightarrow F(\omega) = \pi (\delta(\omega - 1) + \delta(\omega + 1))$$

$$g(t) = u(t+1/2) - u(t-1/2) \Rightarrow$$

$$\delta(\omega) = 2 \sin(\omega/2) / \omega = \text{sinc} f$$



$$X(\omega) = \frac{1}{2\pi} F(\omega) \times G(\omega)$$

$$\Rightarrow X(\omega) = \frac{1}{2\pi} \left(2\pi \sin\left(\frac{\omega - \omega_0}{2}\right) / (\omega - \omega_0) + 2\pi \sin\left(\frac{\omega + \omega_0}{2}\right) / (\omega + \omega_0) \right)$$

$$\Rightarrow X(\omega) = \sin\left(\frac{\omega - \omega_0}{2}\right) / (\omega - \omega_0) + \sin\left(\frac{\omega + \omega_0}{2}\right) / (\omega + \omega_0)$$

4) İttiliriz Özelliği

$$f(t) \leftrightarrow F(\omega) \quad f(t) \leftrightarrow f^*(t)$$

$$F(t) \leftrightarrow 2\pi f(-\omega) \quad F(t) \leftrightarrow f^*(-f)$$

5) Ölçeklendirme Özelliği

$$f(\alpha t) \leftrightarrow \frac{1}{|\alpha|} F(\omega/\alpha)$$

$$* f(t) \leftrightarrow F(\omega) \Rightarrow f(-t) \leftrightarrow F(-\omega)$$

6) Türev Özelliği

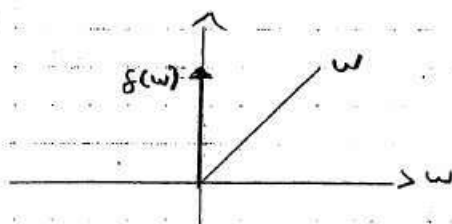
$$\frac{df(t)}{dt} \leftrightarrow (j\omega) \cdot F(\omega) \quad \frac{d^n f(t)}{dt^n} \leftrightarrow (j\omega)^n \cdot F(\omega)$$

Example: $x(t) = u(t) \Rightarrow \delta(\omega)$ 'yi türev özelliğinden yararlanarak bulunuz.

$$\delta(t) = \frac{dx(t)}{dt} \Rightarrow \delta(\omega) = F\left\{ \int_{-\infty}^{\infty} \delta(t) dt \right\} = (j\omega) \cdot F(\omega)$$

$$\Rightarrow \delta(\omega) = j\omega \left(\frac{1}{j\omega} + \pi \delta(\omega) \right) = 1$$

$$* \omega \delta(\omega) = 0$$



7) İntegral Özelliği

$$F\left\{ \int_{-\infty}^t f(z) dz \right\} \leftrightarrow \frac{1}{j\omega} + \pi F(0) \delta(\omega)$$

$$\Rightarrow F(0) = \int_{-\infty}^{\infty} f(t) dt$$

Example: $x(t) = u(t) \Rightarrow x(\omega)$ 'yi Fourier dönüşümünün integral özelliğinden yararlanarak bulun

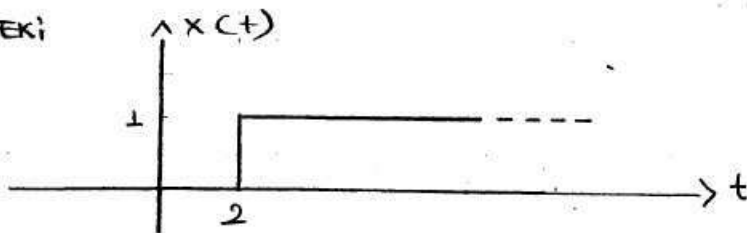
$$x(t) = \int_{-\infty}^{+\infty} \delta(z) \cdot dz \quad \delta(t) = f(t)$$

$$\Rightarrow X(\omega) = \frac{F(\omega)}{j\omega} + \pi F(0) \delta(\omega)$$

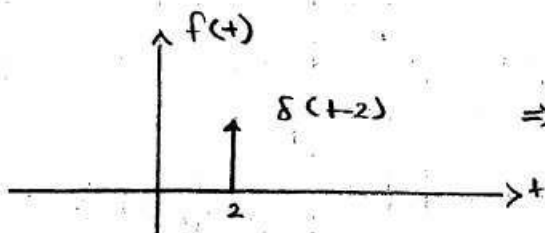
$$f(t) = \delta(t) \Rightarrow F(\omega) = 1 ; F(0) = \int_{-\infty}^{\infty} \delta(t) dt = 1$$

$$\Rightarrow X(\omega) = \frac{1}{j\omega} + \pi \delta(\omega)$$

ÖRNEKİ



$x(\omega)$ 'yi integral özelliğinden yararlanarak bulalım

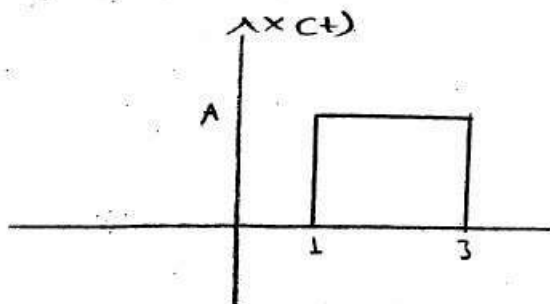


$$\Rightarrow x(t) = \int_2^{\infty} f(t) dt \quad * F(\omega) = e^{-j2\omega}$$

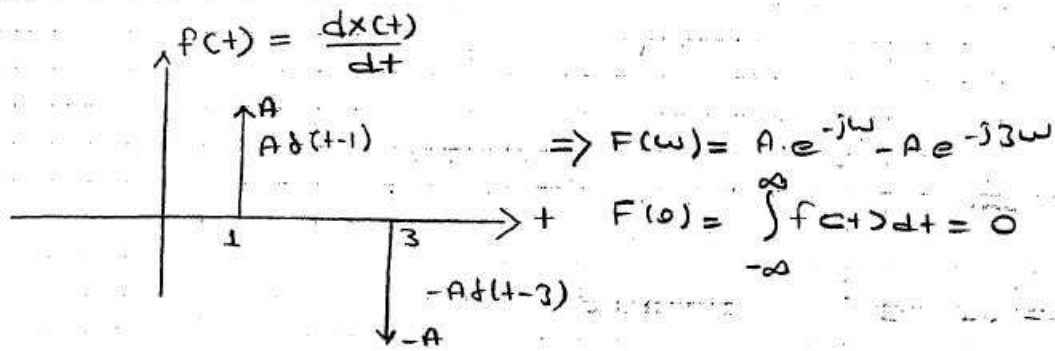
$$\Rightarrow X(\omega) = \frac{F(\omega)}{j\omega} + \pi \underbrace{F(0)}_1 \delta(\omega)$$

$$\Rightarrow X(\omega) = \frac{e^{-j2\omega}}{j\omega} + \pi \delta(\omega) = \frac{-j}{\omega} \times e^{-j2\omega} + \pi \delta(\omega)$$

ÖRNEKİ



$x(\omega)$ 'yi ille olarak integral özelliğinden yararlanarak bulalım;

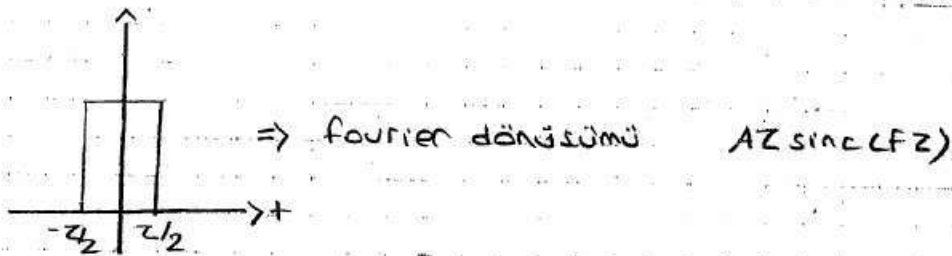


$$\Rightarrow x(\omega) = \frac{F(\omega)}{j\omega} + \pi F(0) \delta(\omega) \Rightarrow x(\omega) = \frac{A}{j\omega} (e^{-j\omega} - e^{-j3\omega})$$

$$\Rightarrow x(\omega) = \frac{A}{j\omega} e^{-j2\omega} (e^{j\omega} - e^{-j\omega})$$

$$\Rightarrow x(\omega) = 2A \cdot e^{-j2\omega} \sin \omega / \omega \Rightarrow x(\omega) = 2A e^{-j2\omega} \cdot \text{sinc}(2f)$$

* İkinci yol;



* Şimdi $x(t)$ 'yi buna benzetmeye çalışalım;

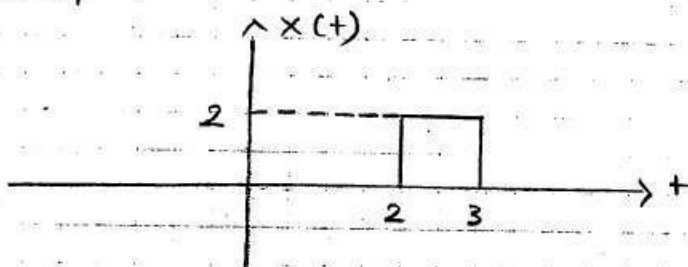
$$Z=2$$

$$A=A$$

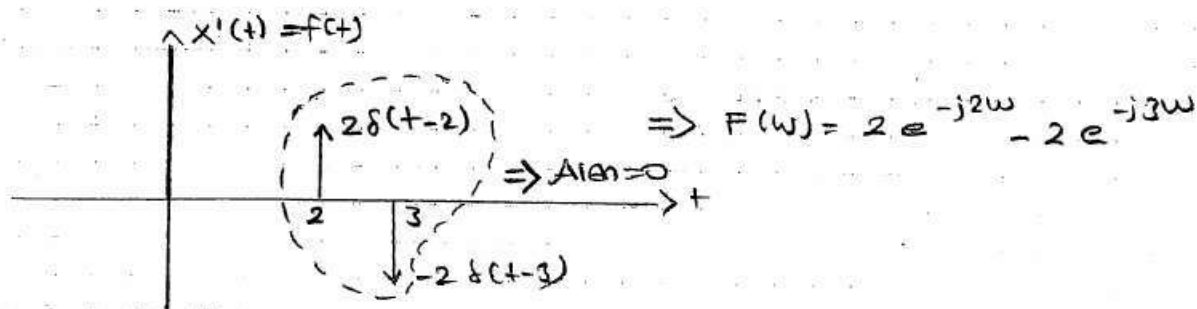
Kayma miktarı: 2

$$\Rightarrow x(\omega) = 2A e^{-j2\omega} \text{sinc}(2f)$$

ÖRNEK;



$x(\omega)$ 'yi integral özelliğinden yararlanarak bulalım;



$$X(\omega) = \frac{F(\omega)}{j\omega} + \pi F(0) \delta(\omega)$$

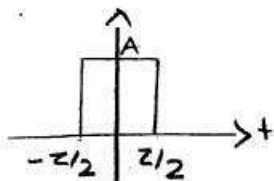
$$\Rightarrow X(\omega) = (2e^{-j2\omega} - 2e^{-j3\omega}) / j\omega$$

$$\Rightarrow X(\omega) = \frac{2}{j\omega} e^{-j2.5\omega} (e^{j0.5\omega} - e^{-j0.5\omega})$$

$$\Rightarrow X(\omega) = \frac{4}{\omega} e^{-j2.5\omega} \sin 0.5\omega$$

$$\Rightarrow X(\omega) = \frac{2}{\pi f} e^{-j2.5\omega} \sin \pi f \Rightarrow X(\omega) = 2 \cdot e^{-j2.5\omega} \cdot \text{sinc}(f)$$

* İkinci yol



$$Az \cdot \text{sinc}(fz)$$

$$z = 1$$

$$A = 2$$

$$\text{Kayma miktarı} = 2.5$$

$$\left. \begin{array}{l} z = 1 \\ A = 2 \\ \text{Kayma miktarı} = 2.5 \end{array} \right\} X(\omega) = e^{-j2.5\omega} \cdot 2 \text{sinc}(f)$$

* Üçüncü yol (Formülden)

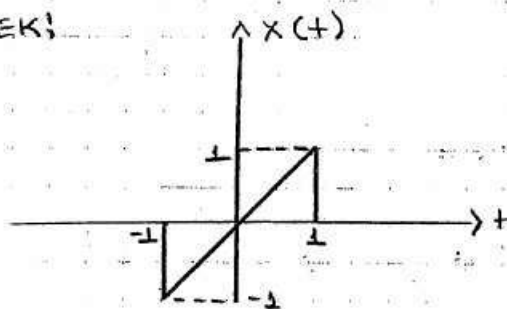
$$\Rightarrow X(\omega) = 2 \int_{-z/2}^{z/2} e^{-j\omega t} dt = \frac{-2}{j\omega} e^{-j\omega t} \Big|_{-z/2}^{z/2}$$

$$\Rightarrow X(\omega) = \frac{-2}{j\omega} (e^{-j3\omega} - e^{-j2\omega})$$

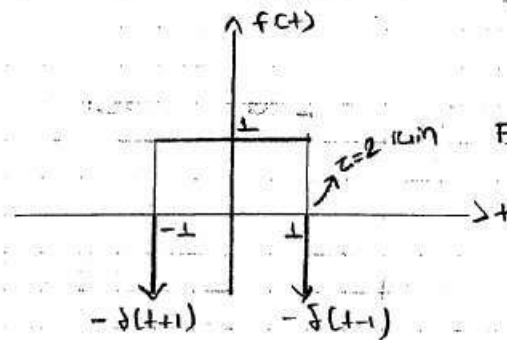
$$\Rightarrow X(\omega) = \frac{2}{j\omega} e^{-j2.5\omega} (e^{j0.5\omega} - e^{-j0.5\omega})$$

$$\Rightarrow X(\omega) = \frac{4}{\omega} e^{-j2.5\omega} \sin \omega/2 \Rightarrow X(\omega) = 2 e^{-j2.5\omega} \text{sinc}(f)$$

ÖRNEKİ



$x(\omega)$ 'yi integral özelliğinden yararlanarak bulalım.



$$F(\omega) = 2 \operatorname{sinc}(2f) - e^{-j\omega} - e^{j\omega}$$

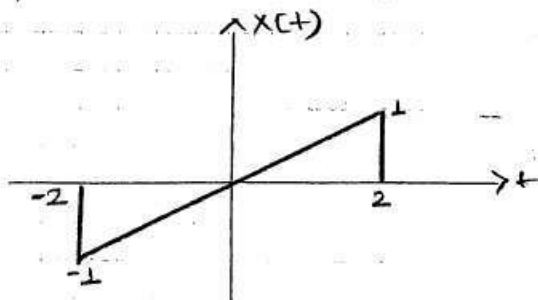
$$\text{Alan} = 0$$

$$\Rightarrow F(\omega) = \frac{2 \sin(\omega)/\omega}{j\omega} - (e^{j\omega} + e^{-j\omega})/j\omega$$

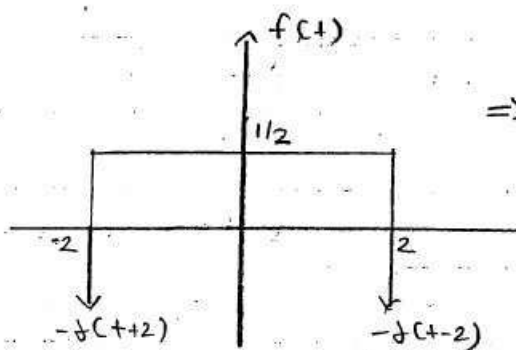
$$\Rightarrow F(\omega) = \frac{2 \sin(\omega)}{j\omega^2} - \frac{2}{j\omega} \cos \omega$$

* Aynı sonuç formülünde çıkar!

ÖRNEKİ



$x(\omega)$ 'yi integral özelliğinden yararlanarak bulalım;



$$\Rightarrow F(\omega) = 2 \operatorname{sinc}(4f) - e^{-j2\omega} - e^{j2\omega}$$

$$\text{Alan} = 0$$

$$\Rightarrow x(\omega) = \frac{\sin(2\omega)/\omega}{j\omega} - (e^{j2\omega} + e^{-j2\omega})/j\omega$$

$$\Rightarrow x(\omega) = \frac{\sin(2\omega)}{j\omega^2} - \frac{2}{j\omega} \cos(2\omega)$$

* formüllerden gözörüm;

$$X(\omega) = \int_{-2}^2 +1/2 \cdot e^{-j\omega t} dt ; +1/2 = u \Rightarrow \frac{dt}{2} = du$$

$$e^{-j\omega t} dt = dv \Rightarrow v = \frac{-1}{j\omega} e^{-j\omega t}$$

$$\Rightarrow X(\omega) = -\left(\frac{1}{2j\omega} e^{-j\omega t}\right) \Big|_{-2}^2 + \frac{1}{2j\omega} \int_{-2}^2 e^{-j\omega t} dt$$

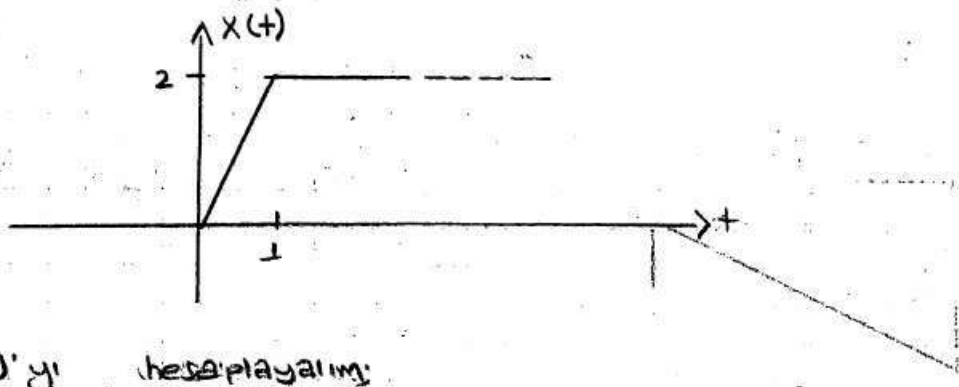
$$\Rightarrow X(\omega) = \frac{-1}{j\omega} (e^{j2\omega} + e^{-j2\omega}) + \frac{1}{2j\omega^2} (e^{-j2\omega} - e^{j2\omega})$$

$$\Rightarrow X(\omega) = \frac{-2}{j\omega} \cos(2\omega) - \frac{1 \times j}{2j\omega^2} (e^{j2\omega} - e^{-j2\omega})$$

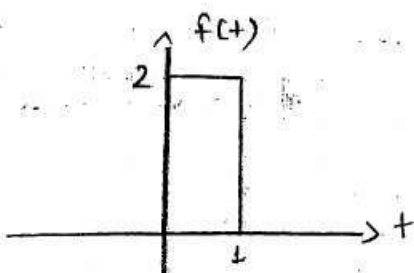
$$\Rightarrow X(\omega) = -\frac{2}{j\omega} \cos(2\omega) - \frac{1}{\omega^2} \sin 2\omega$$

$$\Rightarrow X(\omega) = \frac{\sin 2\omega}{j\omega^2} - 2 \cos(2\omega) / j\omega$$

ÖRNEK;



$x(\omega)$ 'yi hesaplayalım;



$$A \text{ gen} = 2$$

$$A = 2$$

$$Z = 1$$

$$\text{kıyma miktari} = 1/2$$

$$F(\omega) = 2 \text{sinc}(f) \cdot e^{-j\omega/2}$$

$$\Rightarrow X(\omega) = (4 \sin(\omega/2) / \omega \cdot e^{-j\omega/2}) / j\omega + 2\pi \delta(\omega)$$

$$\Rightarrow X(\omega) = \frac{4}{j\omega^2} \sin(\omega/2) \cdot e^{-j\omega/2} + 2\pi \delta(\omega)$$

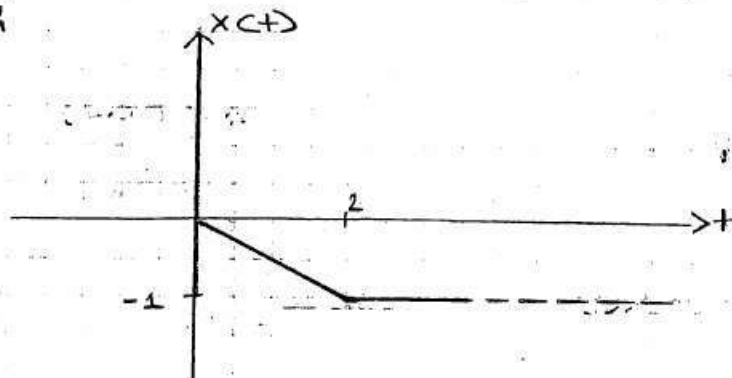
$$\Rightarrow x(\omega) = \frac{4}{j\omega^2} \frac{(e^{j\omega/2} - e^{-j\omega/2})}{2j} e^{-j\omega/2} + 2\pi\delta(\omega)$$

$$\Rightarrow x(\omega) = \frac{-2}{\omega^2} (1 - e^{-j\omega}) + 2\pi\delta(\omega)$$

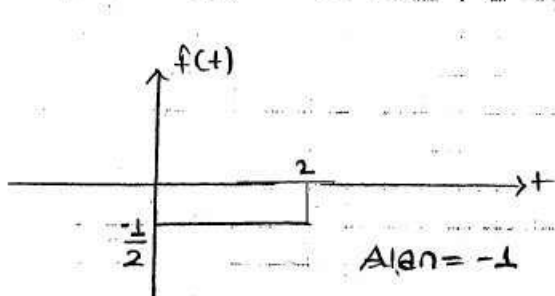
* formüldeolar

$$\Rightarrow x(\omega) = \frac{-2}{\omega^2} + \frac{2e^{-j\omega}}{\omega^2} + 2\pi\delta(\omega)$$

ÖRNEK:



$x(\omega)$ 'yi bulalım



$$A = -1/2$$

$$T = 2$$

Kayma miktarı 1

$$\Rightarrow F(\omega) = -2A \text{sinc}(2f) \cdot e^{-j\omega}$$

$$\Rightarrow x(\omega) = (-2A \sin(\omega)/\omega \times e^{-j\omega}) / j\omega - \pi\delta(\omega)$$

$$\Rightarrow x(\omega) = \frac{-2A}{j\omega^2} \sin\omega \times e^{-j\omega} - \pi\delta(\omega)$$

* formüldeolar

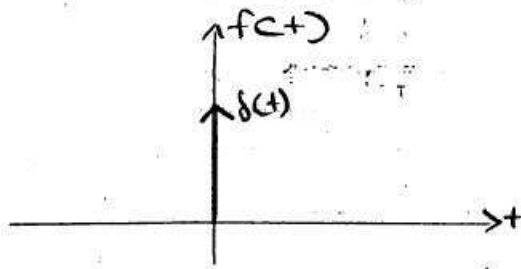
$$\Rightarrow x(\omega) = \frac{-2A}{j\omega^2} \frac{(e^{j\omega} - e^{-j\omega})}{2j} e^{-j\omega} - \pi\delta(\omega) \quad x(\omega) = -1/2 \int_0^2 e^{-j\omega t} dt$$

$$\Rightarrow x(\omega) = \frac{A}{\omega^2} (1 - e^{-j2\omega}) - \pi\delta(\omega)$$

$$\Rightarrow x(\omega) = \frac{-1}{2\omega^2} e^{-j2\omega} + \frac{1}{2\omega^2} - \pi\delta(\omega) \quad \text{bulunur}$$

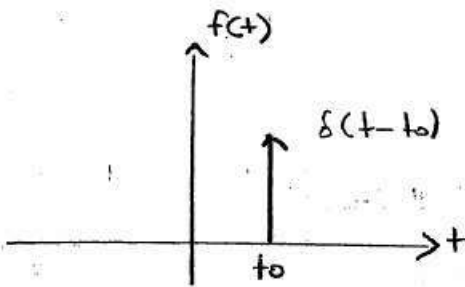
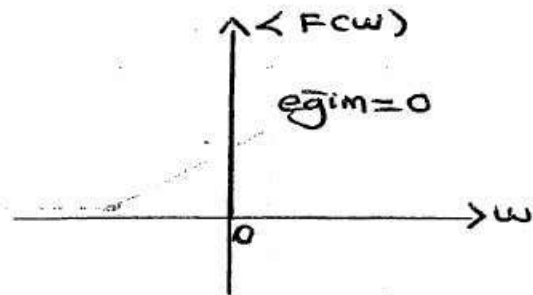
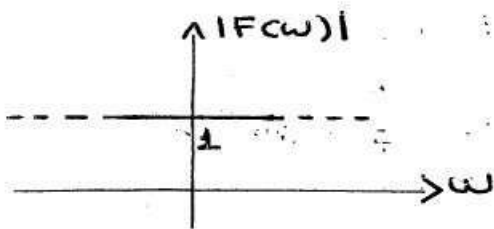
* Bazı önemli fonksiyonların fourier dönüşümleri

• Birim Yürüş İşareti



$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \Rightarrow$$

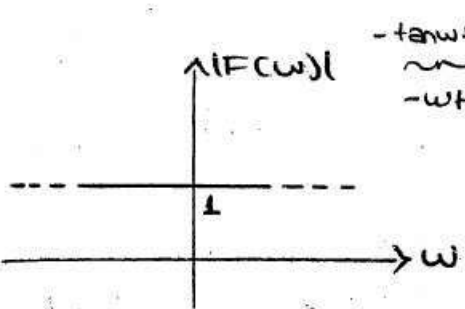
$$F(\omega) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\omega t} dt = 1$$



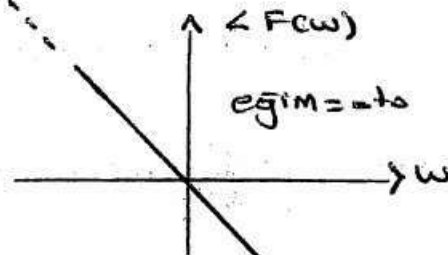
$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt$$

$$F(\omega) = \int_{-\infty}^{\infty} \delta(t - t_0) \cdot e^{-j\omega t} dt$$

$$\Rightarrow F(\omega) = e^{-j\omega t_0} \Rightarrow |F(\omega)| = 1$$



$$\begin{aligned} & -\tan \omega t_0 \quad \left\{ \begin{array}{l} -\frac{\sin \omega t_0}{\cos \omega t_0} \\ -\omega t_0 \end{array} \right. \quad \left\{ \begin{array}{l} \cos \omega t_0 - j \sin \omega t_0 \end{array} \right. \end{aligned}$$

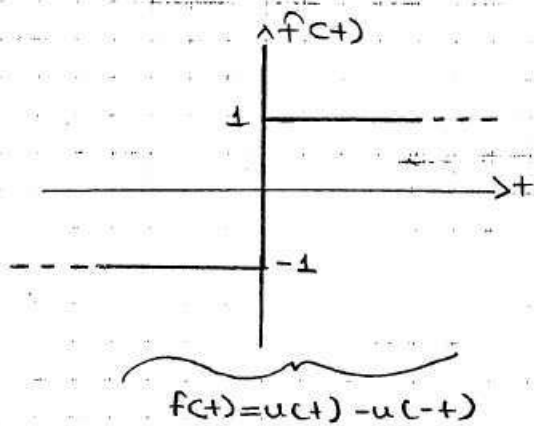


• 1'in fourier dönüşümü

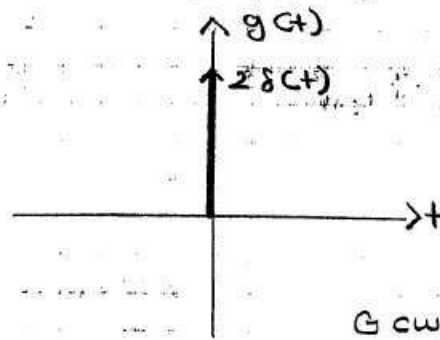
$$1 = e^{j\omega t} \quad (\omega_0 = 0)$$

$$2\pi \delta(\omega - \omega_0) \Rightarrow F\left\{ \int_{-\infty}^{\infty} 1 dt \right\} = \int_{-\infty}^{\infty} e^{-j\omega t} dt = 2\pi \delta(\omega) = \delta(f)$$

• $\text{sgn}(t)$ 'nin Fourier dönüşümü



$F(\omega)$ 'yi integral özelliğinden yararlanarak bulalım;



$$\Rightarrow f'(t) = \delta(t) + \delta(t) = 2\delta(t) = g(t)$$

$$* \omega \delta(\omega) = 0 !$$

$$\Rightarrow F(\omega) = \frac{G(\omega)}{j\omega} + \pi G(0) \delta(\omega) = \frac{2}{j\omega} + 2\pi \delta(\omega) = \frac{2}{j\omega}$$

• $\cos(\omega_0 t)$ 'nin Fourier dönüşümü

$$* \cos(\omega_0 t) = (e^{j\omega_0 t} + e^{-j\omega_0 t}) / 2 = f(t)$$

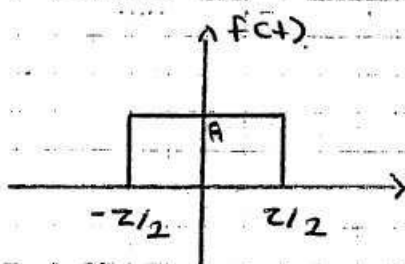
$$F(\omega) = (2\pi \delta(\omega - \omega_0) + 2\pi \delta(\omega + \omega_0)) / 2 = \pi (\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$$

• $\sin(\omega_0 t)$ 'nin Fourier dönüşümü

$$* \sin(\omega_0 t) = (e^{j\omega_0 t} - e^{-j\omega_0 t}) / 2j = f(t)$$

$$F(\omega) = (2\pi \delta(\omega - \omega_0) - 2\pi \delta(\omega + \omega_0)) / 2j = \frac{\pi}{j} (\delta(\omega - \omega_0) - \delta(\omega + \omega_0))$$

• Dikdörtgen Vurus İzaeti



$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt$$

$$\Rightarrow F(\omega) = \int_{-\tau/2}^{\tau/2} A \cdot e^{-j\omega t} dt = \frac{-A}{j\omega} e^{-j\omega t} \Big|_{-\tau/2}^{\tau/2}$$

$$\Rightarrow F(\omega) = \frac{-A}{j\omega} (e^{-j\omega \tau/2} - e^{j\omega \tau/2}) = \frac{A}{j\omega} (e^{j\omega \tau/2} - e^{-j\omega \tau/2})$$

$$\Rightarrow F(\omega) = \frac{2A}{\omega} \sin \omega \tau/2 = A \tau \frac{\sin \omega \tau/2}{\omega \tau/2} = A \tau \frac{\sin \pi f \tau}{\pi f \tau}$$

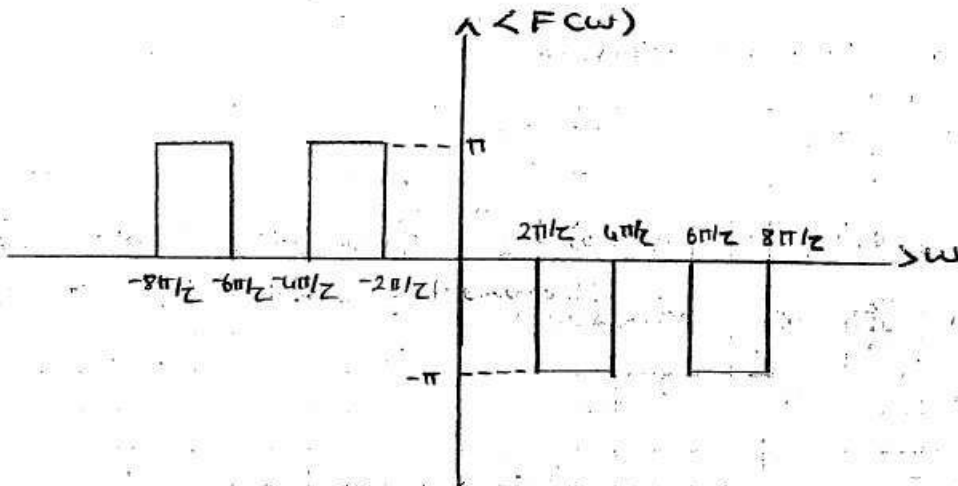
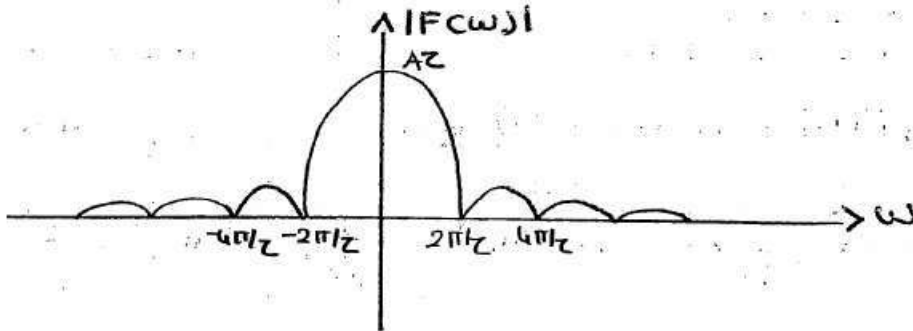
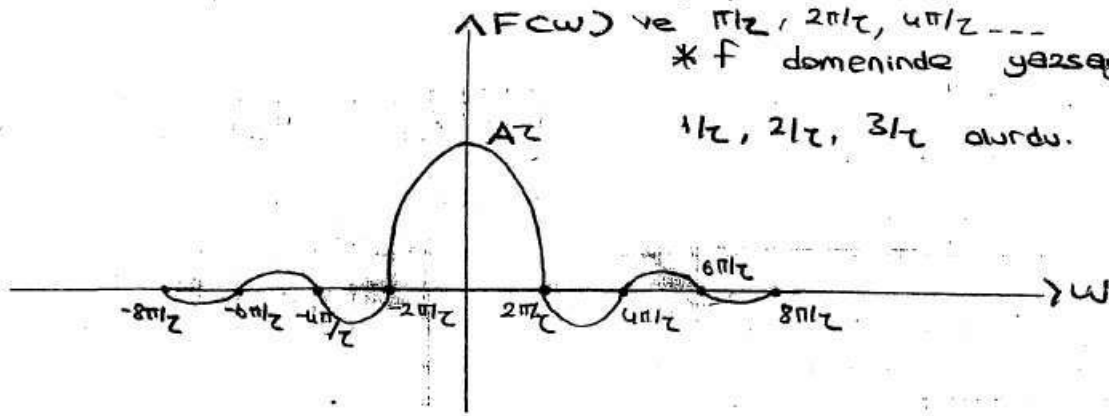
$$\Rightarrow F(\omega) = A \tau \text{sinc}(f \tau) \text{ 'dur.}$$

* $2A\tau \text{sinc}(2f\tau)$

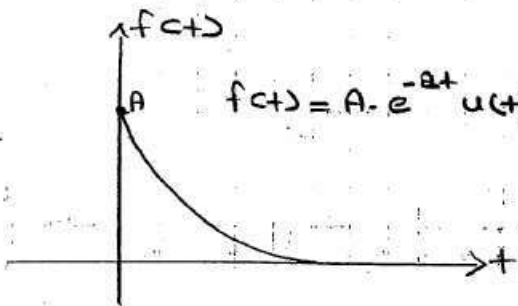
$1/2\tau, 2/2\tau, 4/2\tau, \dots$

* f domeninde yaksaydik

$1/\tau, 2/\tau, 3/\tau$ olurdu.



• Tez Yanlı Üstel Yurus İshareti



$f(t) = A \cdot e^{-at} u(t)$

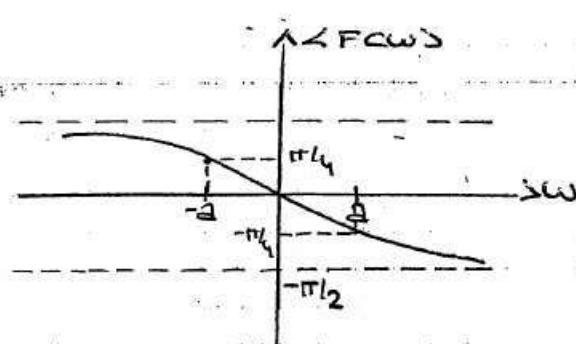
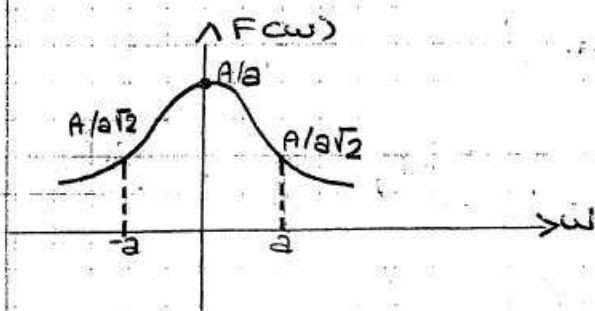
$F(\omega) = \int_0^{\infty} A e^{-at} e^{-j\omega t} dt$

$= A \int_0^{\infty} e^{-(a+j\omega)t} dt$

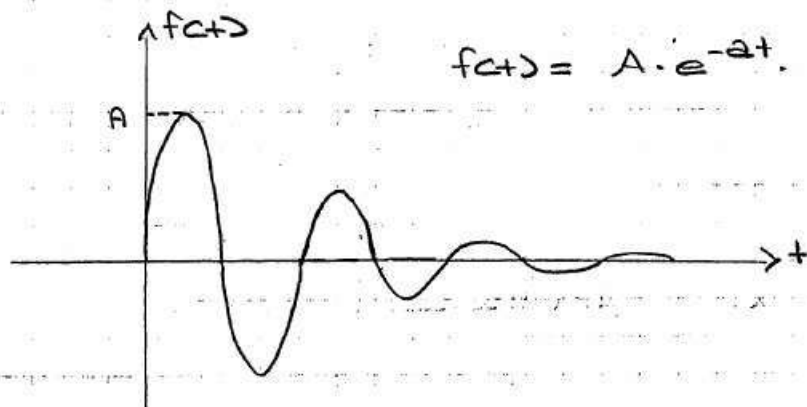
$= \frac{-A}{a+j\omega} e^{-(a+j\omega)t} \Big|_0^{\infty}$

$\Rightarrow F(\omega) = \frac{A}{a+j\omega} = \frac{A(a-j\omega)}{a^2+\omega^2} \quad \angle F(\omega) = -\arctan(\omega/a)$

$\Rightarrow |F(\omega)| = \frac{A}{\sqrt{a^2+\omega^2}}$



• Sönümlü Sinüs işareti



$$f(t) = A \cdot e^{-at} \cdot \sin(\omega_0 t) \cdot u(t)$$

$$F(\omega) = A \int_0^{\infty} e^{-at} \frac{(e^{j\omega t} - e^{-j\omega t})}{2j} e^{-j\omega t} dt$$

$$\Rightarrow F(\omega) = \frac{A}{2j} \int_0^{\infty} e^{-(a-j\omega_0+j\omega)t} dt - \frac{A}{2j} \int_0^{\infty} e^{-(a+j\omega_0+j\omega)t} dt$$

$$\Rightarrow F(\omega) = \frac{A}{2j} \times \frac{(-1)}{a+j(\omega-\omega_0)} \times e^{-(a+j(\omega-\omega_0))t} \Big|_0^{\infty} - \frac{A}{2j} \times \frac{(-1)}{a+j(\omega+\omega_0)} \times e^{-(a+j(\omega+\omega_0))t} \Big|_0^{\infty}$$

$$\Rightarrow F(\omega) = \frac{-A}{2j} \times \frac{1}{a+j(\omega-\omega_0)} \times (-1) + \frac{A}{2j} \times \frac{1}{a+j(\omega+\omega_0)} \times (-1)$$

$$\Rightarrow F(\omega) = \frac{A}{2j} \left(\frac{1}{a+j(\omega-\omega_0)} - \frac{1}{a+j(\omega+\omega_0)} \right)$$

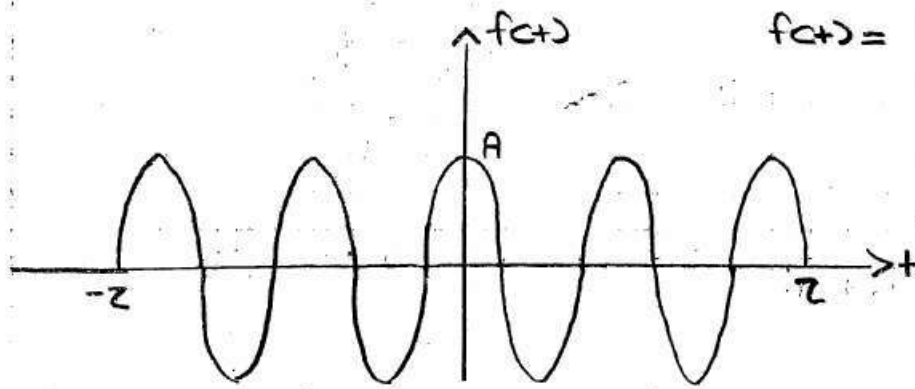
$$\Rightarrow F(\omega) = \frac{A}{2j} \frac{(a+j\omega + j\omega_0 - a - j\omega + j\omega_0)}{a^2 + a j\omega - a j\omega_0 + a j\omega - \omega^2 + \omega_0^2 + a j\omega_0 - j\omega_0 \omega + \omega_0^2}$$

$$\Rightarrow F(\omega) = \frac{A}{2j} \frac{(2j\omega_0)}{a^2 + 2a j\omega - \omega^2 + \omega_0^2}$$

$$\Rightarrow F(\omega) = \frac{A\omega_0}{a^2 - \omega^2 + \omega_0^2 + 2a j\omega} \Rightarrow |F(\omega)| = \frac{A\omega_0}{\sqrt{(a^2 - \omega^2 + \omega_0^2)^2 + 4a^2\omega^2}}$$

$$\Rightarrow \angle F(\omega) = \arctan \frac{-2a\omega}{a^2 + \omega_0^2 - \omega^2}$$

• Yüksek Frekanslı Uçur İşareti



$$f(t) = \begin{cases} A \cos \omega t & \text{Hız} \\ 0 & \text{diger} \\ & \text{yönlende} \end{cases}$$

Bu işaretin fourier dönüşümü, zamanda çarpma özelliğinden yararlanarakta bulunabilir.

$$x(t) = A \cos \omega t \Rightarrow X(\omega) = A\pi (\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$$

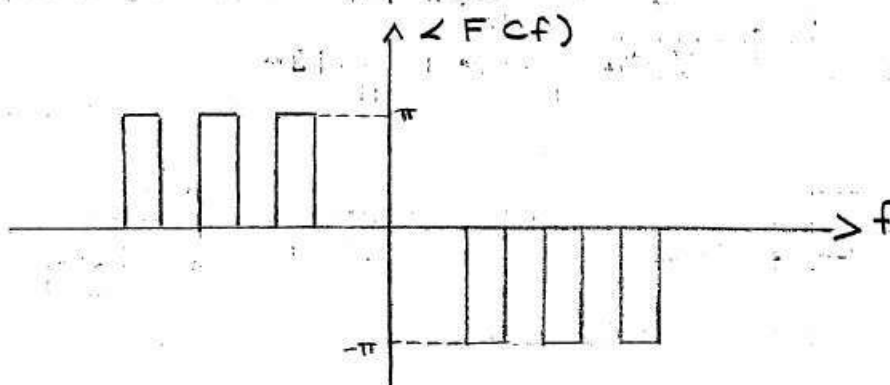
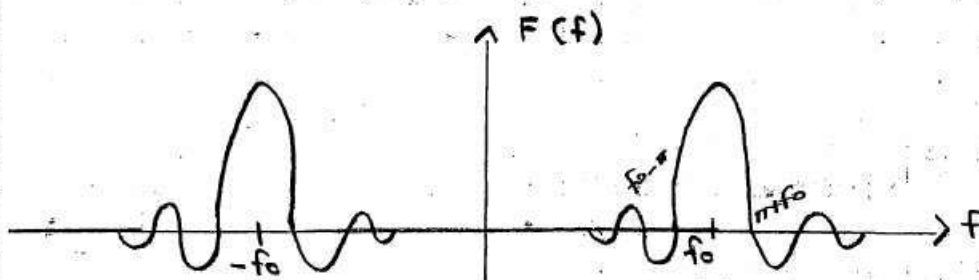
$$g(t) = u(t+z) - u(t-z) \Rightarrow G(\omega) = 2z \operatorname{sinc}(2fz) = 2z \sin(\omega z) / \omega z$$

Burada $A=1$
 $z=2z$

• hâlde $F(\omega)$, bu işareti fourier dönüşümünün katlamasının 2π 'ye bölümüdür.

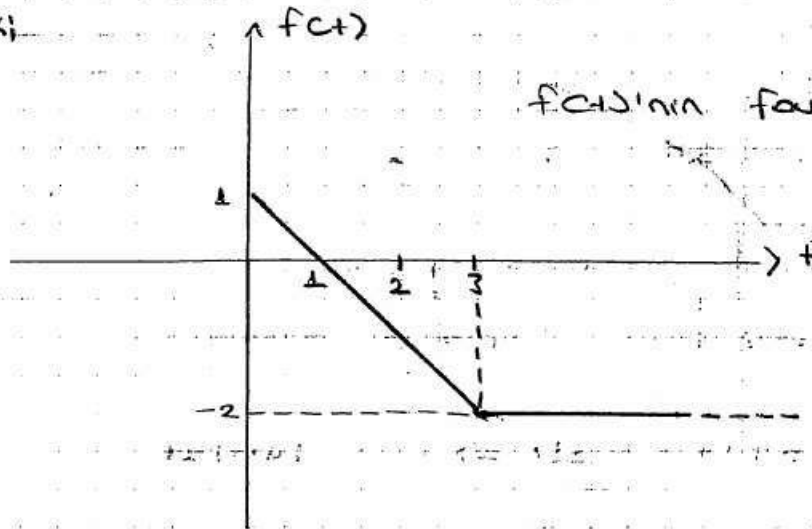
$$F(\omega) = AZ \left(\frac{\sin(\omega - \omega_0)Z}{(\omega - \omega_0)Z} + \frac{\sin(\omega + \omega_0)Z}{(\omega + \omega_0)Z} \right)$$

$$\Rightarrow F(f) = AZ (\operatorname{sinc}(cf - f_0)Z + \operatorname{sinc}(cf + f_0)Z)$$



* Gauss yuruş isaretinin fourier dönüşümünde bir gauss'tur.

ÖRNEKİ;

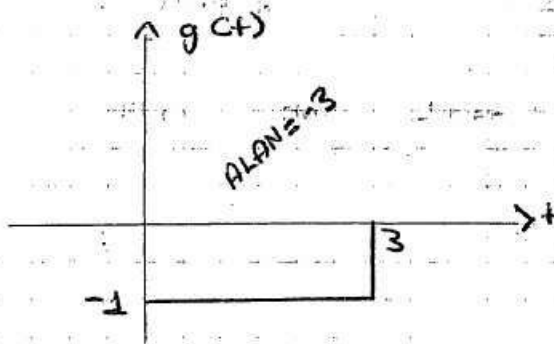


f(t)'nin fourier dönüşümünü bulunuz

* $\int_{-\infty}^{\infty} |f(t)| dt < \infty$ şartı sağlanmadığı için fourier dönüşümü yatınsamaz. Fakat fourier dönüşümünün integral özelliğinden yararlanarak fourier dönüşümünü hesaplayabiliriz.

* $g(t) = f'(t) = \frac{df(t)}{dt}$ olsun.

o hâlde; $F(\omega) = \frac{G(\omega)}{j\omega} + \pi G(0) \delta(\omega)$ olur. (integ.-özel.)



* Şimdi ise fourier dönüşümünü bildiğimiz bir fonksiyon olan dikdörtgen yuruş isaretine benzetmeye çalışarak fourier dönüşümünü hesaplayalım:

$$A = -1$$

$$T = 3$$

Kayma miktarı 1.5

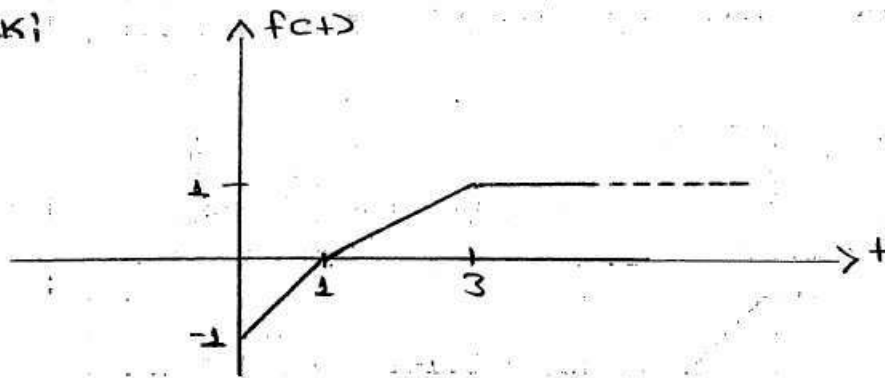
$$\Rightarrow G(\omega) = -3 \operatorname{sinc}(3f) \cdot e^{-j3\omega/2} = -3 \frac{\sin 3\pi f}{3\pi f} \times e^{-j3\omega/2}$$

$$\Rightarrow G(\omega) = -\frac{2}{\omega} \sin(3\omega/2) \times e^{-j3\omega/2}$$

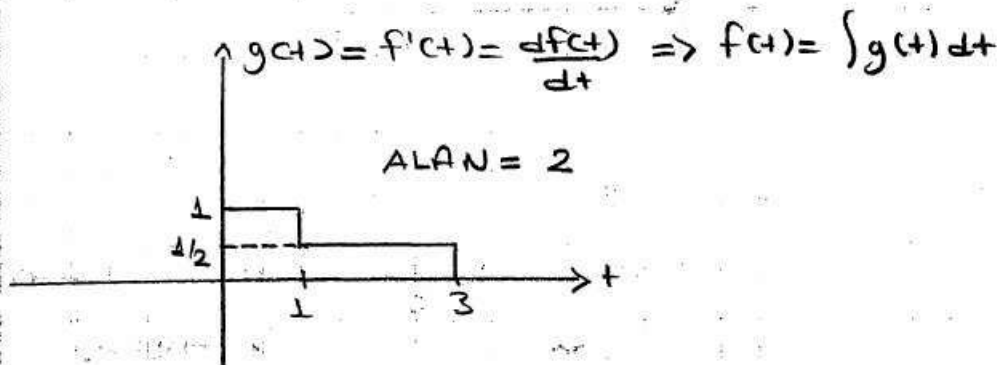
$$\Rightarrow G(\omega) = \frac{-2}{2j\omega} (e^{j3\omega/2} - e^{-j3\omega/2}) e^{-j3\omega/2}$$

$$\Rightarrow G(\omega) = \frac{-2}{2j} 1 + \frac{2}{2j} e^{-j3\omega} \Rightarrow F(\omega) = \frac{1}{\omega^2} - \frac{1}{\omega^2} e^{-j3\omega} - 3\pi \delta(\omega)$$

ÖRNEKİ



$F(\omega)$ 'yi hesaplayalım; (Integral özelliğiyle)



0-1 (cm) $A=1, z=1, k.m=1/2 \Rightarrow e^{-j\omega/2} \text{sinc}(f)$

1-3 (cm) $A=1/2, z=2, k.m=2 \Rightarrow e^{-j2\omega} \text{sinc}(2f)$

$$\Rightarrow X_1(\omega) = e^{-j\omega/2} \frac{\sin \pi f}{\pi f} = \frac{2e^{-j\omega/2}}{\omega} \times \sin(\omega/2)$$

$$\Rightarrow X_2(\omega) = e^{-j2\omega} \frac{\sin 2\pi f}{2\pi f} = \frac{e^{-j2\omega}}{\omega} \times \sin(\omega)$$

$$\Rightarrow X_1(\omega) + X_2(\omega) = \frac{2}{\omega} \times \frac{e^{-j\omega/2}}{2j} \times (e^{j\omega/2} - e^{-j\omega/2}) + \frac{e^{-j2\omega}}{\omega} \times \frac{1}{2j} (e^{j\omega} + e^{-j\omega})$$

$$\Rightarrow F(\omega) = \frac{X_1(\omega) + X_2(\omega)}{j\omega} + 2\pi\delta(\omega)$$

$$\Rightarrow F(\omega) = \frac{2}{2j\omega} \times \frac{1}{j\omega} \times 1 + \frac{2}{2j\omega} \times \frac{1}{j\omega} e^{-j\omega} + \frac{1}{2j\omega} \times \frac{1}{j\omega} e^{-j\omega} - \frac{e^{-j3\omega}}{2j\omega} \times \frac{1}{j\omega} + 2\pi\delta(\omega)$$

$$\Rightarrow F(\omega) = -1/\omega^2 + \frac{1}{2\omega^2} e^{-j\omega} + \frac{1}{2\omega^2} e^{-j3\omega} + 2\pi\delta(\omega)$$

* formülde aynı sonucu buluruz.

$$G(\omega) = \int_0^1 e^{-j\omega t} dt + \frac{1}{2} \int_1^3 e^{-j\omega t} dt$$

$$\Rightarrow G(\omega) = \left[\frac{1}{j\omega} e^{-j\omega t} \right]_0^1 - \frac{1}{2j\omega} \left[e^{-j\omega t} \right]_1^3 = \frac{1}{j\omega} e^{-j\omega} + \frac{1}{j\omega} - \frac{1}{2j\omega} e^{-j3\omega} + \frac{1}{2j\omega} e^{-j\omega}$$

$$\Rightarrow \frac{G(\omega)}{j\omega} = \frac{e^{-j\omega}}{\omega^2} - \frac{1}{\omega^2} + \frac{1}{2\omega^2} e^{-j3\omega} - \frac{1}{2\omega^2} e^{-j\omega}$$

$$\Rightarrow F(\omega) = \frac{-1}{\omega^2} + \frac{1}{2\omega^2} e^{-j\omega} + \frac{1}{2\omega^2} e^{-j3\omega} + 2\pi \delta(\omega) \text{ dir.}$$

8) İnti Özelliği

$$C_{fg}(z) = \int_{-\infty}^{\infty} g(t) \cdot f(t-z) dt = \int_{-\infty}^{\infty} f(t) \cdot g(t+z) dt \rightarrow G(\omega) \cdot F(-\omega) \\ \leftrightarrow G(f) \cdot F(-f)$$

* Periyodik olmayan işaretlerin genlik ve faz tayfı sürekli.

* Periyodik işaretlerin fourier dönüşümü hesaplanamaz.

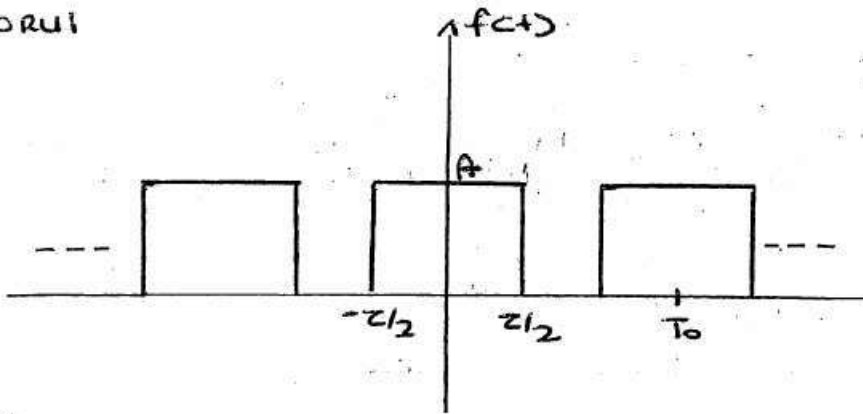
fazat dolayı olarak hesaplanabilir. Bu işaretlerin genlik ve faz tayfı çizgiden oluşur.

$$x(t) = e^{+j\omega_0 t} \Rightarrow X(\omega) = 2\pi \delta(\omega - \omega_0) \Rightarrow X(f) = \delta(f - f_0)$$

$$x(t) = e^{+j2\omega_0 t} \Rightarrow X(\omega) = 2\pi \delta(\omega - 2\omega_0) \Rightarrow X(f) = \delta(f - 2f_0)$$

SORULAR

7. SORU



Fourier dönüşümünü hesaplayınız.

* Fourier serisi katsayıları ile Fourier dönüşümü arasındaki ilişki:

$$F_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} f(t) e^{-jk\omega_0 t} dt = \frac{1}{T_0} F(k\omega_0)$$

periyot! $\omega_0 = 2\pi/T_0$

Bu işaretin Fourier dönüşümü $F(\omega) = A T_0 \text{sinc}(\omega T_0/2)$

$$\text{Yani } F(\omega) = A T_0 \frac{\sin(\omega T_0/2)}{\omega T_0/2} \Rightarrow F(\omega) = 2 A T_0 \sin(\omega T_0/2) / \omega$$

$$\text{O hâldde } F_k = \frac{2 A T_0}{T_0} \sin(\pi k \omega_0 T_0/2) / k \omega_0$$

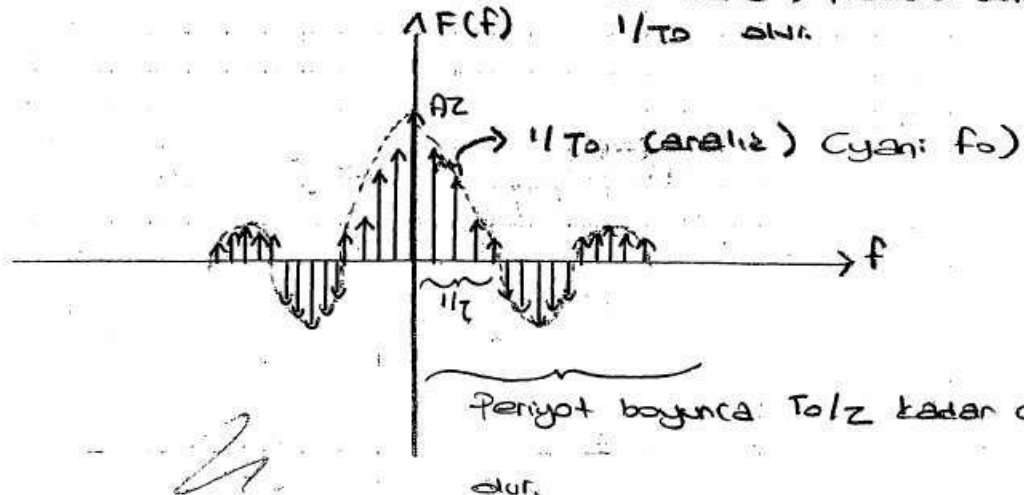
$$\Rightarrow F_k = \frac{2 A T_0}{T_0 2 \pi k \omega_0} \sin(\pi k \omega_0 T_0/2) \Rightarrow F_k = \frac{A T_0}{T_0} \text{sinc}(k \omega_0 T_0/2) \text{ olur.}$$

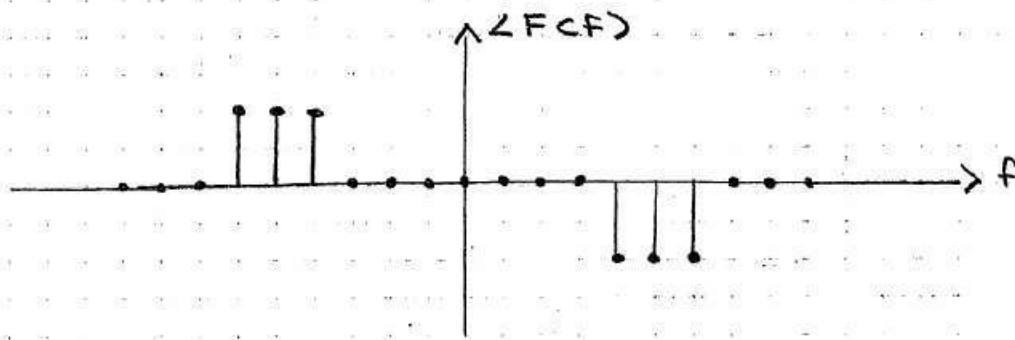
$$\text{O hâldde } f(t) = \sum_{k=-\infty}^{\infty} \frac{A T_0}{T_0} \text{sinc}(k \omega_0 T_0/2) e^{jk \omega_0 t}$$

$$\Rightarrow F(f) = \sum_{k=-\infty}^{\infty} \frac{A T_0}{T_0} \text{sinc}(k \omega_0 T_0/2) \delta(f - k \omega_0/2\pi) \text{ olur.}$$

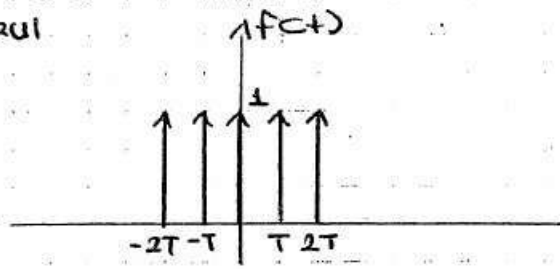
burada δ dir
yapısal değeri
kT_0 dir.

$T_0 \rightarrow 2\pi \Rightarrow$ zaman domaininde
 T_0 ileler; frekans domaininde
 $1/T_0$ olur.





? SORU!



Fourier dönüşümüne bakalım!

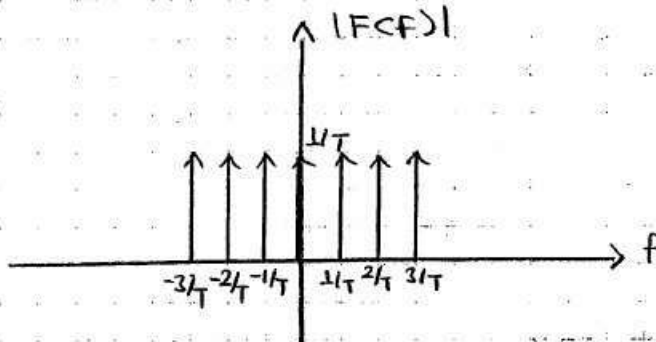
$$f(t) = \sum_{k=-\infty}^{\infty} F_k \cdot e^{jk\omega_0 t} \Rightarrow F_k = \frac{1}{T_0} \int_{T_0} f(t) \cdot e^{-jk\omega_0 t} dt$$

$$\Rightarrow F_k = \frac{1}{T} \int_0^T \delta(t) \cdot e^{-jk2\pi/T \cdot t} dt = \frac{1}{T}$$

* $\omega_0 = 2\pi/T$ dir.

O hâlde $f(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{jk\omega_0 t}$

$$F(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta(f - k f_0) \Rightarrow F(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta(f - k \frac{1}{T}) \text{ olur.}$$



* faz tayfı 0 olur.

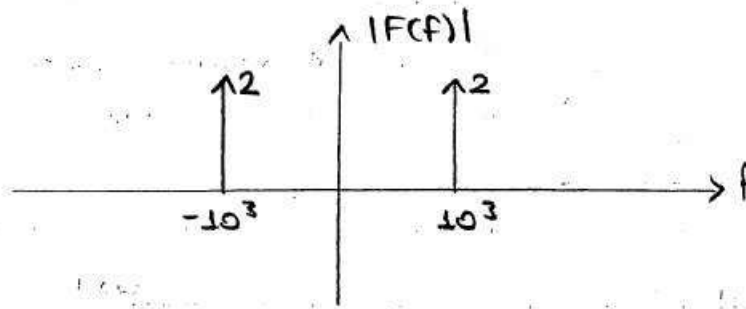
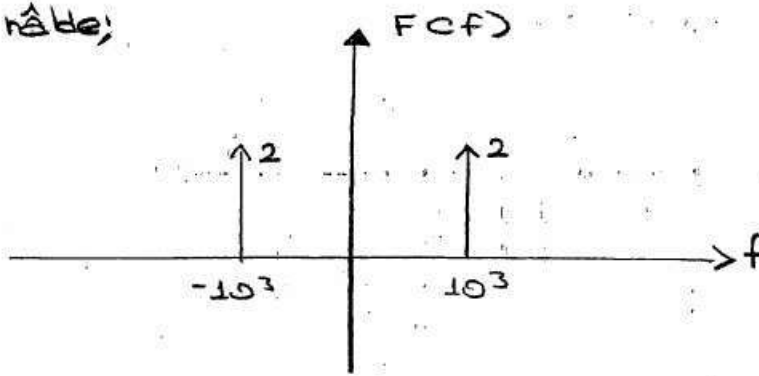
SORU! $f(t) = 4 \sin[2\pi 10^3 t - \pi/3]$ Fourier dönüşümünü hesaplayınız.

$$f(t) = \frac{4}{2j} (e^{j(2\pi 10^3 t - \pi/3)} - e^{-j(2\pi 10^3 t - \pi/3)})$$

$$\Rightarrow F(f) = \frac{4}{2j} e^{-j\pi/3} \delta(f - 10^3) - \frac{4}{2j} e^{j\pi/3} \delta(f + 10^3)$$

$$\Rightarrow F(f) = -2j e^{-j\pi/3} \delta(f - 10^3) + 2j e^{j\pi/3} \delta(f + 10^3)$$

O hâle;



Şimdi de faz tayfini çizelim

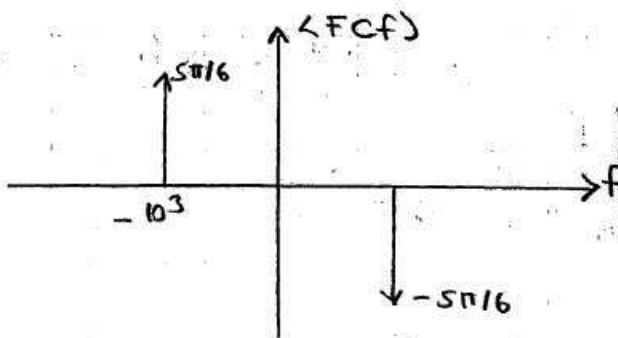
$$0-j \Rightarrow \arctan(-1/0) = \arctan(-\infty) \Rightarrow \theta = -\pi/2$$

$$0+j \Rightarrow \arctan(+1/0) = \arctan(+\infty) \Rightarrow \theta = \pi/2$$

$$e^{-j\pi/3} \rightarrow -\pi/3 \text{ ve } e^{j\pi/3} \rightarrow \pi/3 \Rightarrow -\pi/3 - \pi/2 = -5\pi/6$$

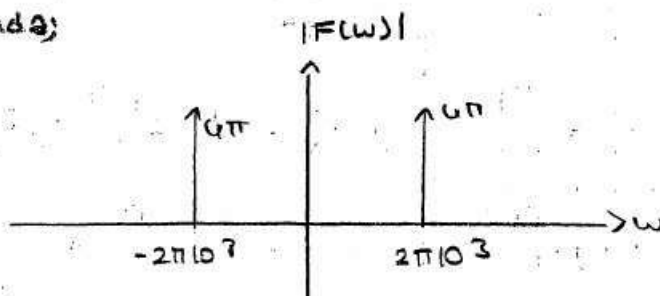
$$\pi/3 + \pi/2 = 5\pi/6$$

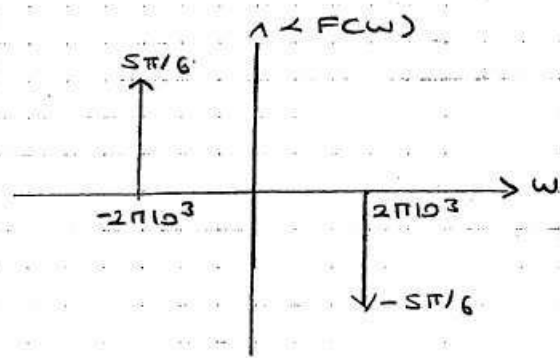
Yani: payda: j + kısım için -90° olarak yazarı alır
- kısım için 90° olarak yazarı alır.



$$F(\omega) \text{ 'yı yazalım: } F(\omega) = -u\pi j e^{-j\pi/3} \delta(f-10^3) + u\pi j e^{j\pi/3} \delta(f+10^3)$$

Bu durumda;



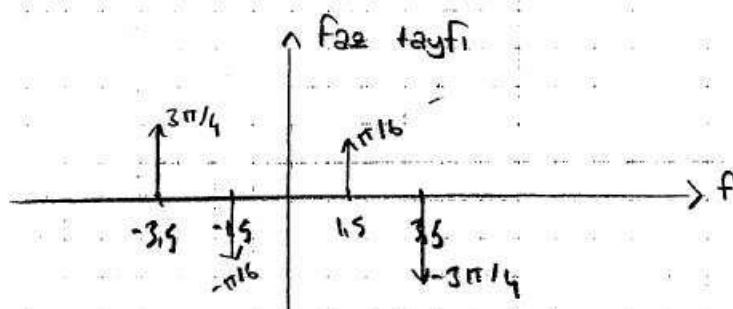
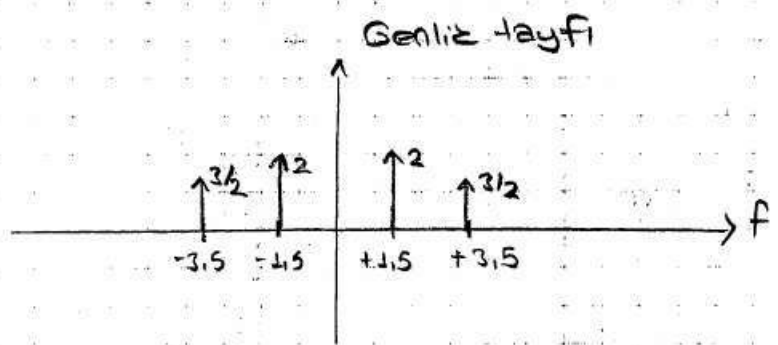


SORU: $x(t) = 3 \sin(7\pi t - \pi/4) - 4 \cos(3\pi t + \pi/6)$ min genlik ve faz tayfini çiziniz.

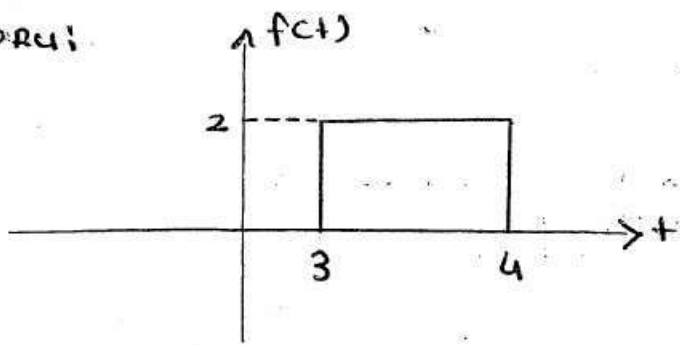
$$x(t) = \frac{3}{2j} e^{j(7\pi t - \pi/4)} - e^{-j(7\pi t - \pi/4)} - 2 \left(e^{j(3\pi t + \pi/6)} + e^{-j(3\pi t + \pi/6)} \right)$$

$w = 7\pi$ $w = 7\pi$ $f = 3/2$ $f = -3/2$
 $f = 3,5$ $f = -3,5$

$$x(t) = \frac{3}{2} (-j) e^{j(7\pi t - \pi/4)} + \frac{3}{2} (1j) e^{-j(7\pi t - \pi/4)} - 2 e^{j(3\pi t + \pi/6)} + 2 e^{-j(3\pi t + \pi/6)}$$



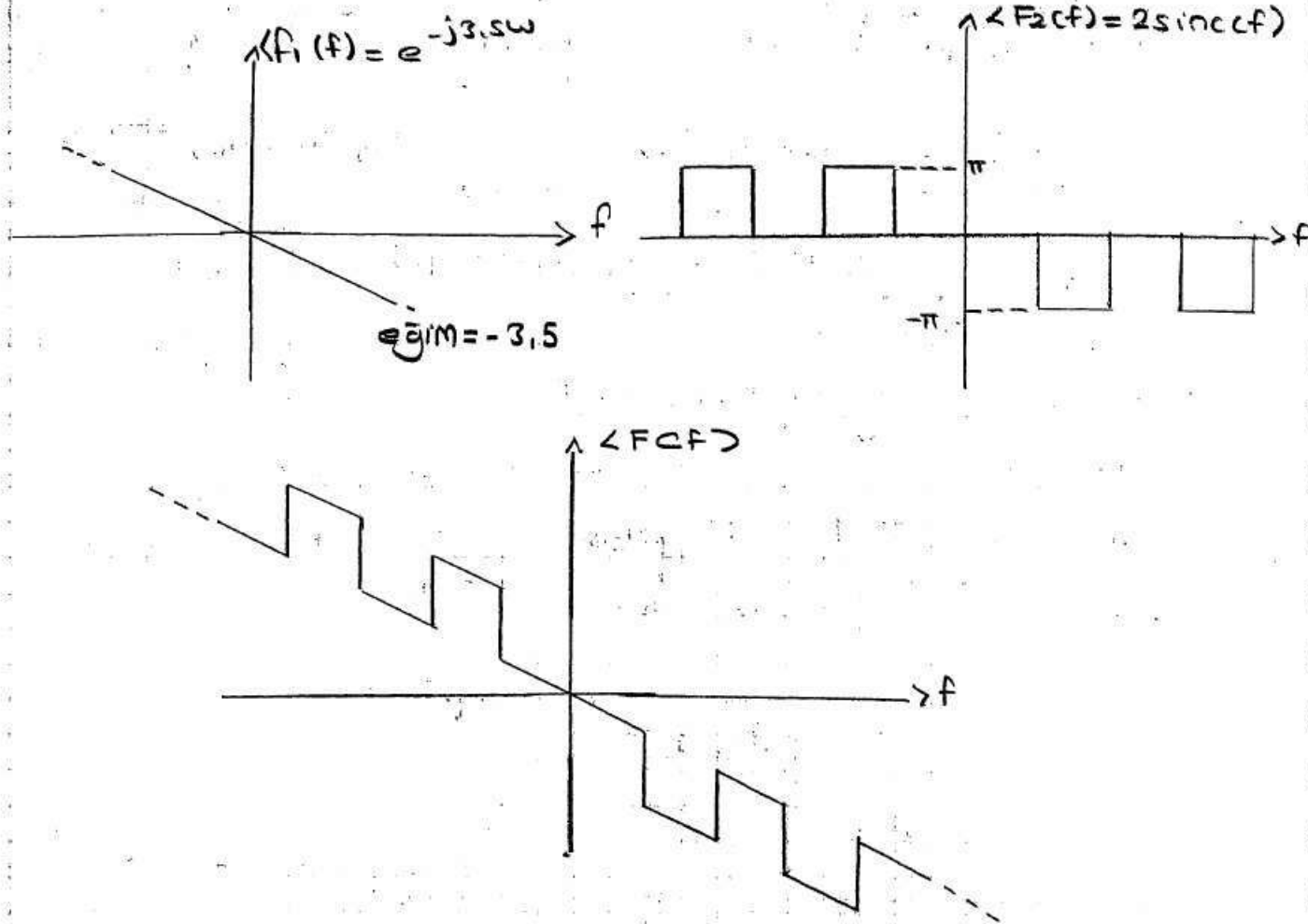
SORU:



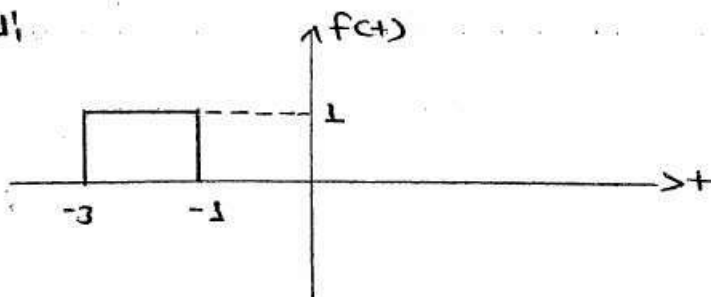
$$F(\omega) = ?$$

$$\left. \begin{array}{l} A=2 \\ z=1 \\ k.m=3,5 \end{array} \right\} F(\omega) = 2 \operatorname{sinc}(\omega) \cdot e^{-j3,5\omega}$$

* formüllede hesaplanır! (22un yol)



SORU:



$$F(\omega) = ?$$

A1.1
Z1.2
K.M1-2

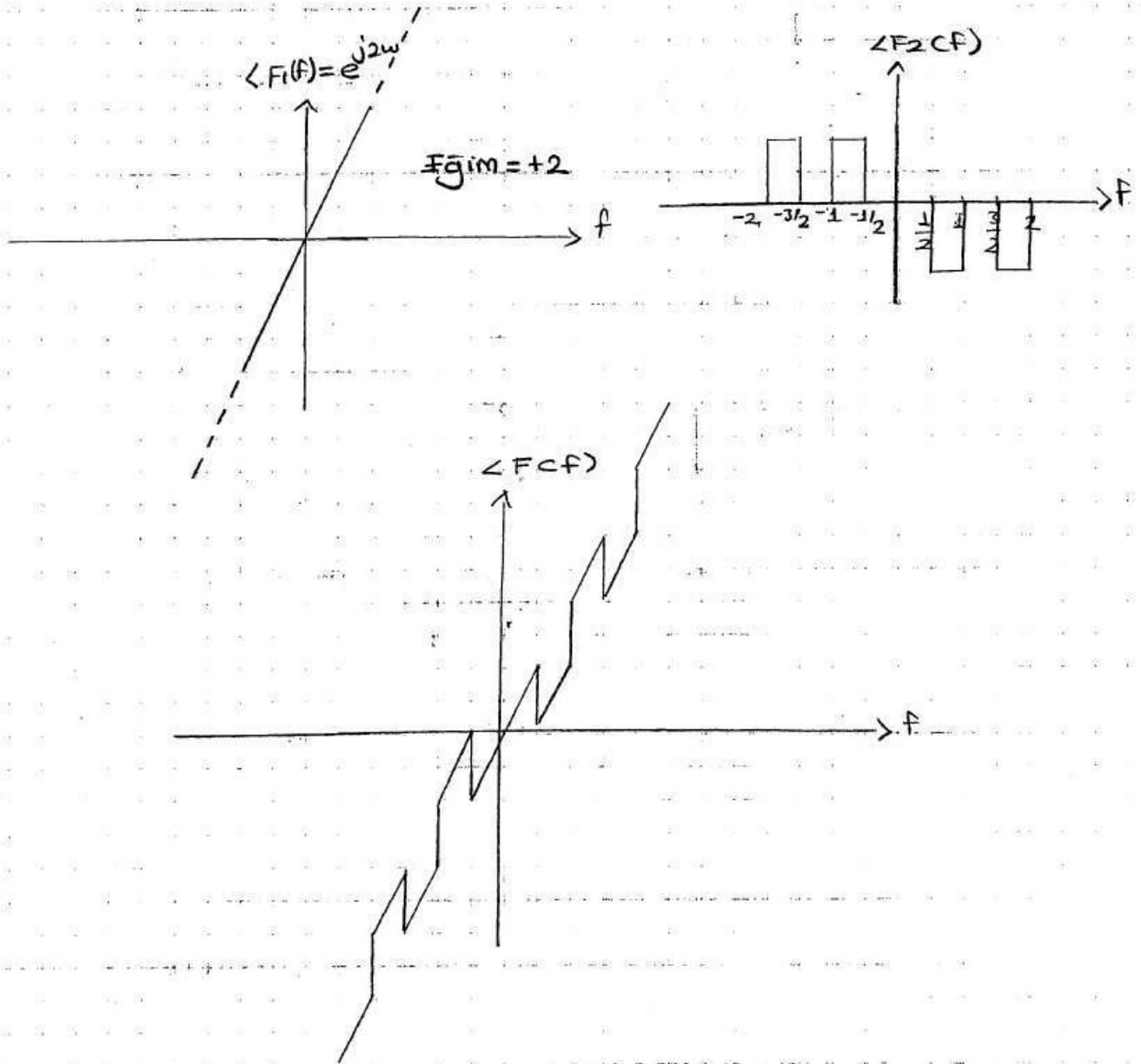
$$\left. \begin{array}{l} A1.1 \\ Z1.2 \\ K.M1-2 \end{array} \right\} F(\omega) = 2 \operatorname{sinc}(2f) \cdot e^{j2\omega}$$

* Formülünde hesaplayalım;

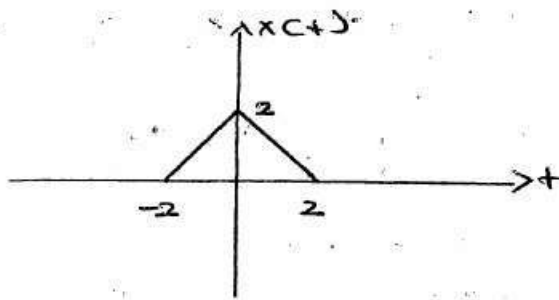
$$F(\omega) = \int_{-3}^1 e^{-j\omega t} dt = \left. \frac{-1}{j\omega} e^{-j\omega t} \right|_{-3}^1 = \frac{-1}{j\omega} (e^{j\omega} - e^{j3\omega})$$

$$\Rightarrow F(\omega) = \frac{-e^{j2\omega}}{j\omega} (e^{-j\omega} - e^{j\omega}) = \frac{2e^{j2\omega}}{2j\omega} (e^{j\omega} - e^{-j\omega})$$

$$\Rightarrow F(\omega) = 2e^{j2\omega} \frac{\operatorname{sinc}(\omega)}{\omega} = 2e^{j2\omega} \operatorname{sinc}(2f)$$

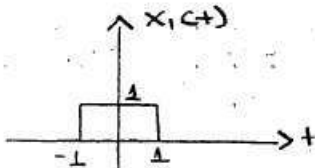


SORU!



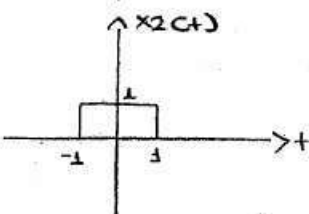
Fourier dönüşümünü hesaplayınız.

* Bu işaret için kalama özelliğini kullanabiliriz.



$$x(t) = x_1(t) * x_2(t)$$

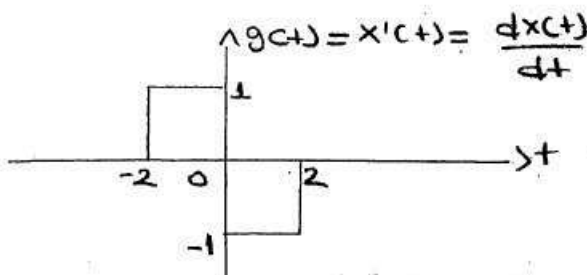
$$\Rightarrow X(\omega) = X_1(\omega) \cdot X_2(\omega) \text{ dir.}$$



$$X_1(\omega) = X_2(\omega) = 2 \operatorname{sinc}(2f)$$

$$\Rightarrow X(\omega) = 4 \operatorname{sinc}^2(2f)$$

* $X(\omega)$ 'yi simdi de integral özelliğiyle gözelim



ALAN=0

$$\begin{array}{l} -2 \rightarrow 0 \text{ için} \\ \left. \begin{array}{l} Z=2 \\ A=1 \\ \text{kim}=-1 \end{array} \right\} 2 \operatorname{sinc}(2f) \cdot e^{j\omega} \end{array}$$

$$\begin{array}{l} 2 \rightarrow 0 \text{ için} \\ \left. \begin{array}{l} Z=2 \\ A=-1 \\ \text{kim}=1 \end{array} \right\} -2 \operatorname{sinc}(2f) \cdot e^{-j\omega} \end{array}$$

$$\Rightarrow X(\omega) = [2 \operatorname{sinc}(2f) (e^{j\omega} - e^{-j\omega})] / j\omega$$

$$\Rightarrow X(\omega) = [2 \operatorname{sinc}(2f) \frac{2}{\omega} \sin(\omega)] = 2 \operatorname{sinc}(2f) \times 2 \operatorname{sinc}(2f)$$

$$\Rightarrow X(\omega) = 4 \operatorname{sinc}^2(2f)$$

SORU: $x(t) = \underbrace{3 \operatorname{sinc}(2t)}_{f(t)} \cdot \underbrace{\cos 2\pi 10^3 t}_{g(t)} \Rightarrow x(\omega)$ yi bulunuz.

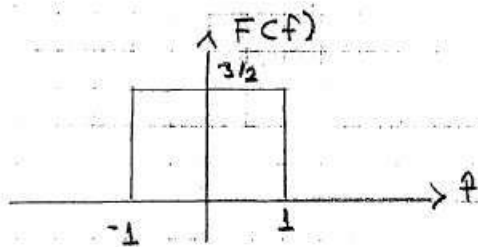
$$x(t) = f(t) \cdot g(t) \Rightarrow X(\omega) = \frac{1}{2\pi} F(\omega) * G(\omega)$$

* Ters Fourier dönüşümünü kullanalım:

$$F(f) = \int_{-B}^B A \cdot e^{j2\pi ft} dt = \frac{A}{j2\pi f} \left[e^{j2\pi ft} \right]_{-B}^B = \frac{A}{j2\pi f} (e^{j2\pi fB} - e^{-j2\pi fB})$$

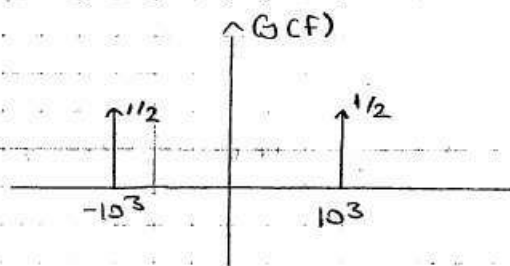
$$\Rightarrow F(f) = \frac{A}{\pi f} \sin 2\pi fB = 2AB \operatorname{sinc}(2Bf)$$

$\downarrow A = 3/2$
 $\downarrow B = 1$

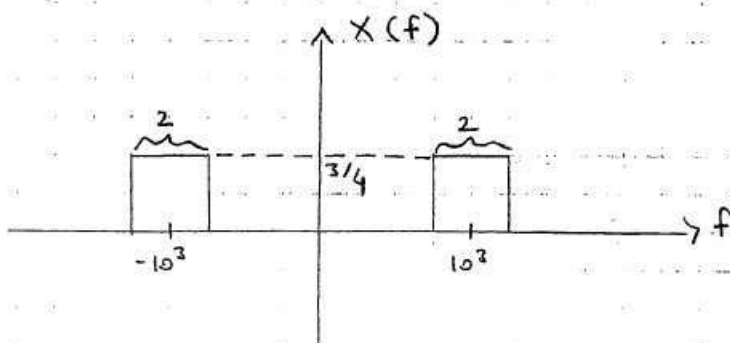


$$g(t) = \cos 2\pi 10^3 t \Rightarrow G(\omega) = \pi (\delta(\omega - 2\pi 10^3) + \delta(\omega + 2\pi 10^3))$$

$$\Rightarrow G(f) = \frac{1}{2} (\delta(f - 10^3) + \delta(f + 10^3))$$



$$\Rightarrow x(f) = F(f) * G(f)$$



SORU! $f(t) \leftrightarrow F(\omega)$

$\Rightarrow \frac{d}{dt} [e^{j2\pi 10^3 t} f(2t-3)]$ -in fourier dönüşümünü hesaplayınız.

* Bu örnekte parantezin içinden başlayarak dışa doğru gideceğiz.

$$f(t) \leftrightarrow F(\omega) \Rightarrow f(2t) \leftrightarrow \frac{1}{2} F(\omega/2)$$

$$\Rightarrow f(2t-3) \leftrightarrow \frac{1}{2} F(\omega/2) e^{-j\omega/2 \times 3}$$

zamanla kayma özelliği!

$$f(t-t_0) \leftrightarrow e^{-j\omega t_0} F(\omega)$$

Şimdi ise modülasyon özelliğini kullanalım;

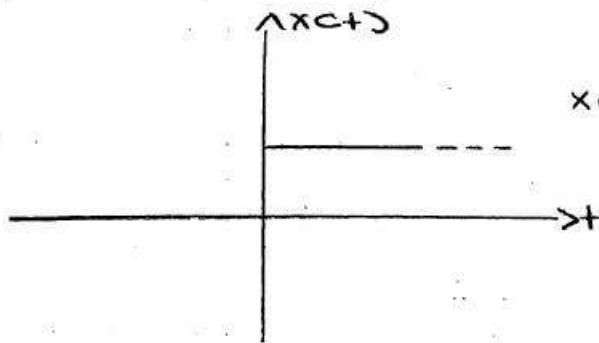
$$e^{j\omega_0 t} f(t) \leftrightarrow F(\omega - \omega_0) \text{ idi.}$$

Burada $\omega_0 = 2\pi 10^3$ 'tür.

O hâlede;

$$\frac{d}{dt} [e^{j2\pi 10^3 t} f(2t-3)] \leftrightarrow (j\omega) \frac{1}{2} F(\frac{\omega - 2\pi 10^3}{2}) \times e^{-j(\frac{\omega - 2\pi 10^3}{2}) \times 3}$$

SORU!



$$x(t) = u(t) \Rightarrow X(\omega) = ?$$

$$u(t) = \int_0^{\infty} \delta(t) dt \text{ dir. } \delta(t) = f(t) \Rightarrow F(\omega) = 1$$

$$\Rightarrow X(\omega) = \frac{1}{j\omega} + \pi F(0) \delta(\omega)$$

$$\Rightarrow X(\omega) = \frac{1}{j\omega} + \pi \delta(\omega)$$

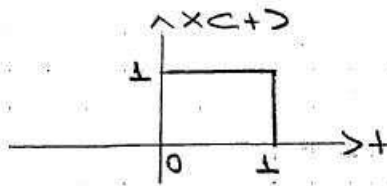
$$\Rightarrow X(f) = \frac{1}{j2\pi f} + \frac{1}{2} \delta(f)$$

Soru: $x(t) = u(t) - u(t-1)$ işareti veriliyor.

a) $x(t)$ 'nin Fourier dönüşümü $X(f)$ 'yi bulunuz.

b) $X(f)$ 'nin iki yanlı genlik tayfı $|X(f)|$ 'yi ölçekli olarak çiziniz.

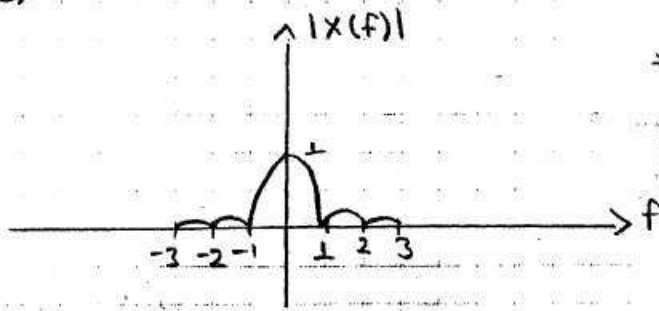
c) $X(f)$ 'nin iki yanlı faz tayfı $\angle X(f)$ 'yi ölçekli olarak çiziniz.



A: 1
Z: 1
Kısm: 1/2

$$2) X(f) = \underbrace{\text{sinc}(cf)}_{\frac{\sin \pi f}{\pi f}} \cdot e^{-j\omega/2}$$

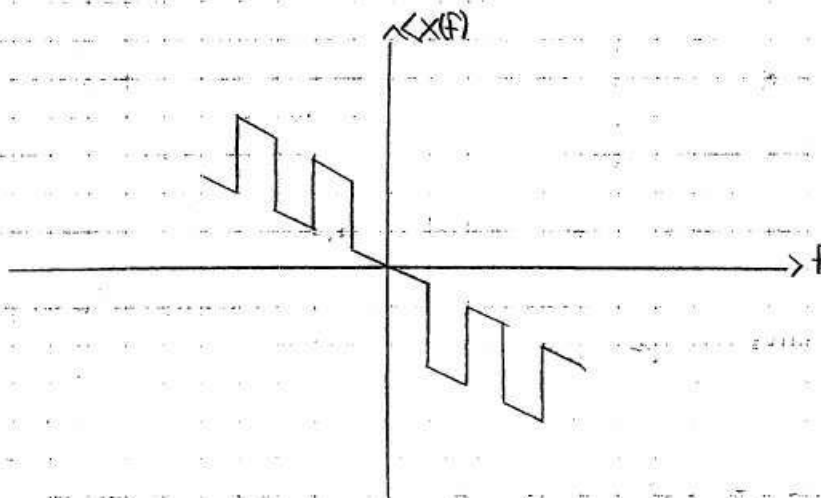
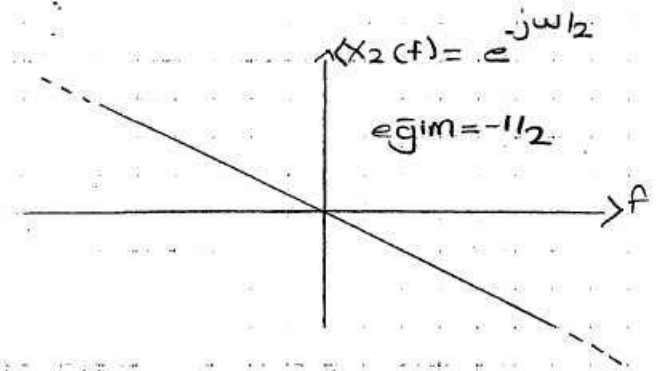
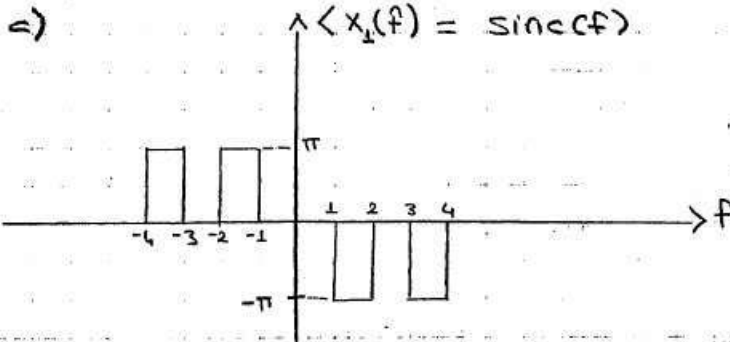
b)



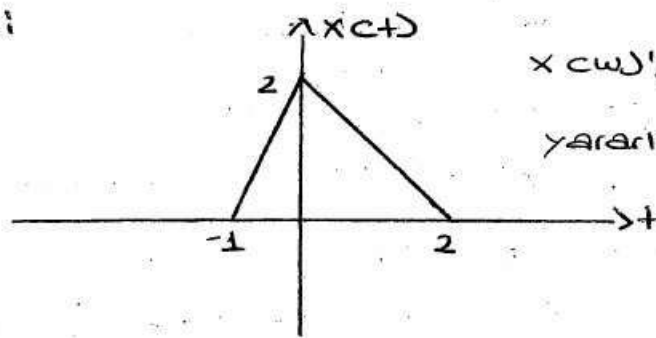
$$* |e^{-j\omega/2}| = 1$$

* Başka bir ifade olaydı tepe değerini değiştirirdi!

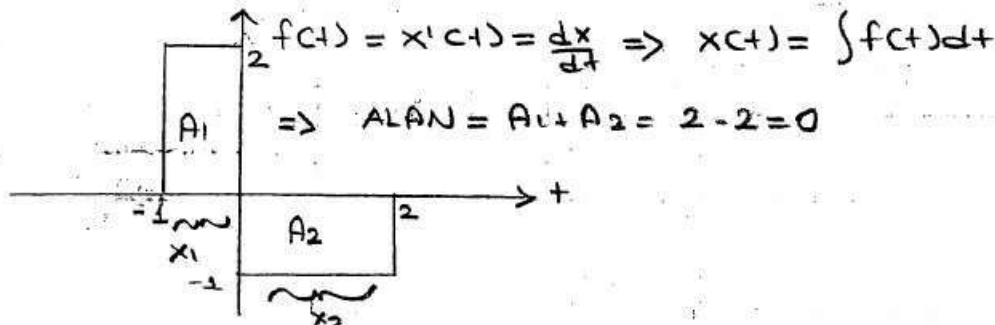
c)



SORU:



$x(t)$ 'yi integral özelliğinden yararlanarak bulun.



$$x_1(\omega) = \int_{-1}^0 2 e^{-j\omega t} dt = \left. \frac{-2}{j\omega} e^{-j\omega t} \right|_{-1}^0 = \frac{-2}{j\omega} + \frac{2}{j\omega} e^{j\omega}$$

$$x_2(\omega) = - \int_0^2 1 e^{-j\omega t} dt = \left. \frac{1}{j\omega} e^{-j\omega t} \right|_0^2 = \frac{1}{j\omega} e^{-j2\omega} - \frac{1}{j\omega}$$

$$\Rightarrow F(\omega) = \frac{-3}{j\omega} + \frac{e^{-j2\omega}}{j\omega} + \frac{2}{j\omega} e^{j\omega}$$

$$\Rightarrow X(\omega) = \frac{3}{\omega^2} - \frac{e^{-j2\omega}}{\omega^2} - \frac{2}{\omega^2} e^{j\omega}$$

SORU: $x(t) = \underbrace{3 \sin(6\pi 10^3 t + \pi/6)}_{x_1(t)} - \underbrace{j \cos(12\pi 10^3 t)}_{x_2(t)} + \underbrace{2}_{x_3(t)} \quad -\infty < t < \infty$
veriliyor

a) $x(t)$ 'nin Fourier dönüşümü $X(f)$ 'yi bulun.

b) İki yarı geniz taşı $|x(f)|$ 'yi ölçerli olarak a2in.

c) İki yarı faz taşı $\angle X(f)$ 'yi ölçerli olarak a2in.

$$a) x_1(t) = \frac{3}{2j} (e^{j(6\pi 10^3 t + \pi/6)} - e^{-j(6\pi 10^3 t + \pi/6)})$$

$$\Rightarrow x_1(t) = \frac{3}{2j} x e^{j\pi/6} x e^{j6\pi 10^3 t} - \frac{3}{2j} x e^{-j\pi/6} x e^{-j6\pi 10^3 t}$$

$$\Rightarrow X_1(f) = \frac{3}{2} (-j) e^{j\pi/6} \delta(f - 3 \times 10^3) + \frac{3}{2} (j) x e^{-j\pi/6} \delta(f + 3 \times 10^3)$$

$$x_2(t) = -\frac{j}{2} \times (e^{j12\pi \times 10^3 t} + e^{-j12\pi \times 10^3 t})$$

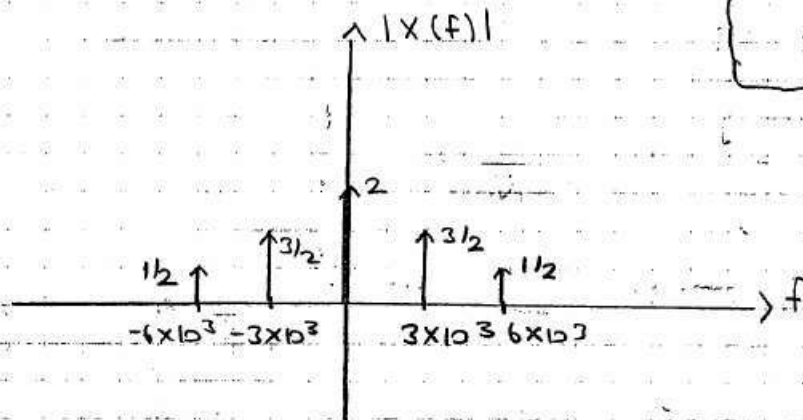
$$\Rightarrow x_2(f) = -\frac{j}{2} \delta(f - 6 \times 10^3) - \frac{j}{2} \delta(f + 6 \times 10^3)$$

$$\Rightarrow x_3(t) = 2 \Rightarrow x_3(f) = 2 \delta(f)$$

o hâlide $x(f) = x_1(f) + x_2(f) + x_3(f)$

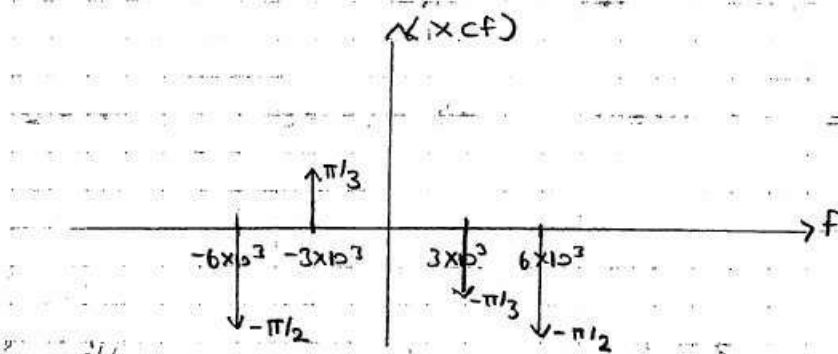
$$\Rightarrow x(f) = \frac{3}{2} (-j) e^{j\pi/6} \delta(f - 3 \times 10^3) + \frac{3}{2} (j) e^{-j\pi/6} \delta(f + 3 \times 10^3) - \frac{j}{2} \delta(f - 6 \times 10^3) - \frac{j}{2} \delta(f + 6 \times 10^3) + 2 \delta(f)$$

b)



$$* 2 e^{j0t} = 2\pi \delta(\omega) = 2 \delta(f)$$

c)



* Ters fourier dönüşümü

$$f(t) = \int_{-\infty}^{\infty} F(f) \cdot e^{j2\pi ft} df$$

$$\Rightarrow x(t) = \int_{-\infty}^{\infty} 2 \delta(f) \cdot e^{j2\pi ft} df = 2 e^{j2\pi t \cdot 0} = 2$$

SORU: Bir sistemin girişi $x(t)$ ve çıkışı $y(t)$

arasındaki ilişki $\frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 2y(t) = x(t)$ diferansiyel denkleminin ile verilmektedir.

a) Sistemin frekans transfer fonksiyonunu bulunuz.

b) Sistemin girişindeki işaret $x(t) = 5 \sin(5t + 20^\circ) + 2 \cos(2t - 50^\circ)$ ise çıkış işaret $y(t)$ 'yi $H(\omega)$ dan yararlanarak bulun.

a) $\frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 2y(t) = x(t)$

$$\Rightarrow (j\omega)^2 y(\omega) + 3(j\omega) y(\omega) + 2y(\omega) = x(\omega)$$

$$\Rightarrow H(\omega) = \frac{y(\omega)}{x(\omega)} = \frac{1}{(j\omega)^2 + 3j\omega + 2}$$

b)

* İlk olarak $x(t)$ 'nin fourier serisi katsayılarını bulalım;

$$x(t) = \frac{5}{2j} \left(e^{j(5t+20^\circ)} - e^{-j(5t+20^\circ)} \right) + e^{j(2t-50^\circ)} + e^{-j(2t-50^\circ)}$$

$$\Rightarrow x(t) = \frac{5}{2j} x e^{j20} x e^{j5t} - \frac{5}{2j} x e^{-j20} x e^{j5t} + e^{-j50} x e^{j2t} + e^{j50} x e^{-j2t}$$

$$\Rightarrow X_2 = \frac{5}{2j} e^{j20}, \omega = 5 \quad \Rightarrow X_1 = e^{-j50}, \omega = 2$$

$$\Rightarrow X_{-2} = \frac{-5}{2j} e^{-j20}, \omega = -5 \quad \Rightarrow X_{-1} = e^{j50}, \omega = -2$$

0 nâbe; $H(s) = \frac{1}{-23+j15} = \frac{-23-j15}{754} = 0,0364 \times e^{-j146,88}$

$$H(-5) = \frac{1}{-23-j15} = \frac{-23+j15}{754} = 0,0364 \times e^{j146,88}$$

$$H(2) = \frac{1}{-2+j6} = \frac{-2-j6}{40} = 0,158 \times e^{-j108,44}$$

$$H(-2) = \frac{1}{-2-j6} = \frac{-2+j6}{40} = 0,158 \times e^{j108,44}$$

$$Y_k = H(j\omega_k) X_k$$

$$\Rightarrow Y_k = \frac{-5}{2j} e^{-j20} \times 0,0364 \times e^{j146,88} + e^{j50} \times 0,158 \times e^{j158,44} \\ + e^{-j50} \times 0,158 \times e^{-j158,44} + \frac{5}{2j} e^{j20} \times 0,0364 \times e^{-j146,88}$$

$$\Rightarrow Y_{-2} = \frac{-0,182}{2j} e^{j126,88}, Y_{-1} = 0,158 e^{j158,44}, Y_1 = 0,158 e^{-j158,44}$$

$$Y_2 = \frac{0,182}{2j} e^{-j126,88}$$

$$\text{ve } y(t) = \sum_{k=-2}^2 Y_k \cdot e^{j\omega_k t} \quad (t \geq 0)$$

$$\Rightarrow y(t) = \frac{-0,182}{2j} e^{j126,88} e^{-jst} + 0,158 e^{j158,44} \times e^{-j2t} + 0,158 \times e^{-j158,44} \\ \times e^{j2t} + \frac{0,182}{2j} \times e^{-j126,88} \times e^{jst}$$

$$\Rightarrow y(t) = 0,182 \times \sin.(st - 126,88) + 0,316 \times \cos.(2t - 158,44)$$

SORU: Bir sistemin birim yuruş tepkisi $h(t) = [e^{-t} - e^{-2t}]u(t)$ 'dir

a) Sistemin frekans transfer fonksiyonunu bulunuz.

b) Sistemin girişindeki işaret $x(t) = 5\sin(t+110^\circ) + 3\cos(2t-40^\circ)$ ise çıkış işareti $y(t)$ 'yi $H(j\omega)$ 'den yararlanarak bulun.

a) Transfer fonksiyonu $H(j\omega)$ 'yi bulalım:

$$h(t) = (e^{-t} - e^{-2t})u(t) \Rightarrow H(j\omega) = \int_0^{\infty} (e^{-t} - e^{-2t}) \cdot e^{-j\omega t} dt$$

$$\Rightarrow H(j\omega) = \int_0^{\infty} e^{-(1+j\omega)t} dt - \int_0^{\infty} e^{-(2+j\omega)t} dt$$

$$\Rightarrow H(j\omega) = \frac{-1}{1+j\omega} e^{-(1+j\omega)t} \Big|_0^{\infty} + \frac{1}{2+j\omega} e^{-(2+j\omega)t} \Big|_0^{\infty}$$

$$\Rightarrow H(j\omega) = \frac{1}{1+j\omega} - \frac{1}{2+j\omega}$$

b) $x(t)$ 'nin fourier serisi katsayılarını bulalım.

$$x(t) = 5 \sin(t + 10^\circ) + 3 \cos(2t - 40^\circ)$$

$$\Rightarrow x(t) = \frac{5}{2j} (e^{j(t+10^\circ)} - e^{-j(t+10^\circ)}) + \frac{3}{2} (e^{j(2t-40^\circ)} + e^{-j(2t-40^\circ)})$$

$$\Rightarrow x(t) = \underbrace{\frac{5}{2j} e^{j10^\circ} \cdot e^{jt}}_{\omega=1} - \underbrace{\frac{5}{2j} e^{-j10^\circ} \cdot e^{-jt}}_{\omega=-1} + \underbrace{\frac{3}{2} e^{-j40^\circ} \cdot e^{j2t}}_{\omega=2} + \underbrace{\frac{3}{2} e^{j40^\circ} \cdot e^{-j2t}}_{\omega=-2}$$

$$y_1 \left\{ \begin{aligned} X_1 &= \frac{5}{2j} e^{j10^\circ}, \omega=1 \\ H(1) &= \frac{1}{1+j} - \frac{1}{2+j} = \frac{1-j}{2} - \frac{2-j}{5} = \frac{1-j3}{10} = 0,316 \times e^{-j71,56} \end{aligned} \right.$$

$$y_{-1} \left\{ \begin{aligned} X_{-1} &= -\frac{5}{2j} e^{-j10^\circ}, \omega=-1 \\ H(-1) &= \frac{1}{1-j} - \frac{1}{2-j} = \frac{1+j}{2} - \frac{2+j}{5} = \frac{1+j3}{10} = 0,316 \times e^{j71,56} \end{aligned} \right.$$

$$y_2 \left\{ \begin{aligned} X_2 &= \frac{3}{2} e^{-j40^\circ}, \omega=2 \\ H(2) &= \frac{1}{1+j2} - \frac{1}{2+j2} = \frac{1-j2}{5} - \frac{2-j2}{8} = \frac{-2-j6}{40} = 0,158 \times e^{-j108,43} \end{aligned} \right.$$

$$y_{-2} \left\{ \begin{aligned} X_{-2} &= \frac{3}{2} e^{j40^\circ}, \omega=-2 \\ H(-2) &= \frac{1}{1-j2} - \frac{1}{2-j2} = \frac{1+j2}{5} - \frac{2+j2}{8} = \frac{-2+j6}{40} = 0,158 \times e^{j108,43} \end{aligned} \right.$$

$$* y_1 = \frac{1,58}{2j} e^{-j61,56}$$

$$* y_{-1} = \frac{-1,58}{2j} e^{j61,56}$$

$$* y_2 = \frac{0,474}{2} e^{-j148,43}$$

$$* y_{-2} = \frac{0,474}{2} e^{j148,43}$$

$$y(t) = \sum_{k=-2}^2 y_k \cdot e^{j\omega_k t} \quad (t \neq 0)$$

o hâlde: $y(t) = \frac{1.58}{2j} e^{-j61.56} e^{jt} - \frac{1.58}{2j} e^{j61.56} e^{-jt} + \frac{0.474}{2} e^{-j148.43} e^{j2t} + \frac{0.474}{2} e^{j148.43} e^{-j2t}$

$\Rightarrow y(t) = 1.58 \sin(t - 61.56) + 0.474 \cos(2t - 148.43^\circ)$

SORU! $x(t) = \sin(2\pi 10^3 t - \pi/6) + u(t) - u(t-1)$, $-\infty < t < \infty$ sinyali veriliyor $x(t)$ -nin fourier dönüşümü $x(f)$ 'yi bulunuz.

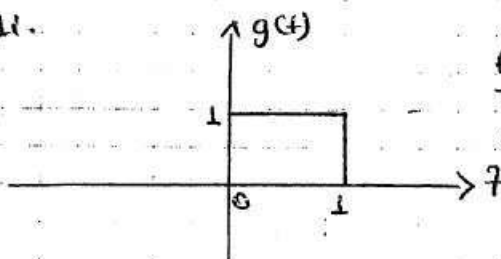
* $x(t) = f(t) + g(t)$ olsun. $f(t) = \sin(2\pi 10^3 t - \pi/6)$, $g(t) = u(t) - u(t-1)$

$f(t) = \left[e^{j(2\pi 10^3 t - \pi/6)} - e^{-j(2\pi 10^3 t - \pi/6)} \right] / 2j$

$\Rightarrow f(t) = \frac{1}{2} (-j) e^{-j\pi/6} e^{j2\pi 10^3 t} + \frac{1}{2} (j) e^{j\pi/6} e^{-j2\pi 10^3 t}$

$\Rightarrow F(f) = \frac{1}{2} (-j) e^{-j\pi/6} \delta(f - 10^3) + \frac{1}{2} (j) e^{j\pi/6} \delta(f + 10^3)$

$g(t) = u(t) - u(t-1)$ idi.



$\left. \begin{array}{l} A: 1 \\ T: 1 \\ K.M: 1/2 \end{array} \right\} G(f) = \text{sinc}(f) \cdot e^{-j\omega/2}$

o hâlde: $X(f) = \frac{1}{2j} e^{-j\pi/6} \delta(f - 10^3) - \frac{1}{2j} e^{j\pi/6} \delta(f + 10^3) + \text{sinc}(f) e^{-j\omega/2}$

SORU! $x(t) = 3\sin(6\pi 10^3 t + \pi/6) + j\cos(12\pi 10^3 t) + 2$ $-\infty < t < \infty$ sinyali veriliyor

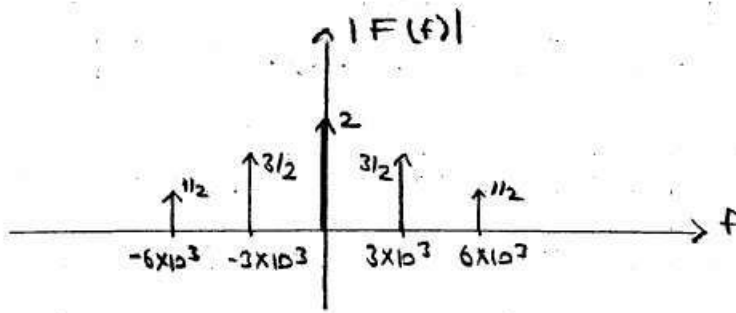
a) $x(t)$ -nin fourier dönüşümü $x(f)$ -i bulunuz.

b) $x(t)$ -nin ortalama gücünü zaman ve frekans domaininde ayrı ayrı hesaplayarak Parseval teoreminin geçerli olduğunu kanıtlayınız.

2) $x(t) = \frac{3}{2j} e^{j(6\pi 10^3 t + \pi/6)} - \frac{3}{2j} e^{-j(6\pi 10^3 t + \pi/6)} + \frac{j}{2} e^{j(12\pi 10^3 t)} + \frac{j}{2} e^{-j(12\pi 10^3 t)} + 2$

$\Rightarrow X(f) = \frac{3}{2j} e^{j\pi/6} \delta(f - 3 \times 10^3) - \frac{3}{2j} e^{-j\pi/6} \delta(f + 3 \times 10^3) + \frac{j}{2} \delta(f - 6 \times 10^3) + \frac{j}{2} \delta(f + 6 \times 10^3) + 2 \delta(f)$

b) zaman domeninde ortalama güç F_z 'ların genlik kareleri toplamıdır. O halde $P_t = \frac{9}{4} + \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + 4 = 9$



$P_f =$ Genlik kareleri toplamı

$$P_f = \frac{9}{4} \times 2 + \frac{1}{4} \times 2 + 4 = 9 \Rightarrow P_t = P_f$$

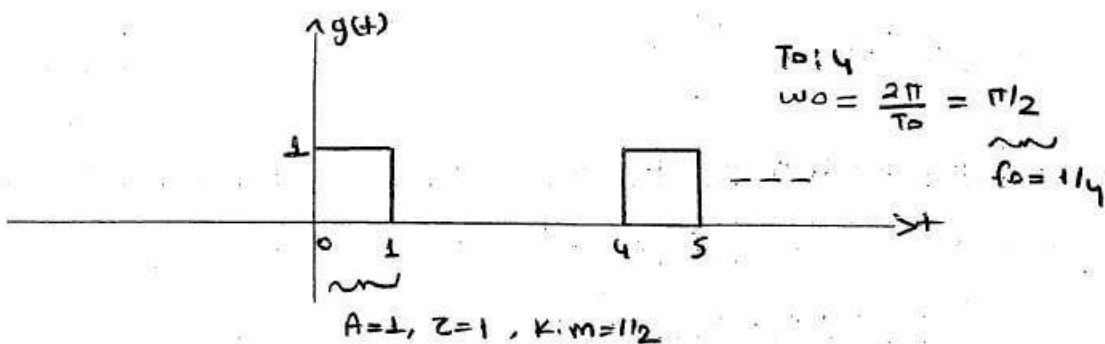
soru: Fourier serisi ile Fourier dönüşümü arasındaki ilişki-
den yararlanarak

a) $g(t) = \sum_{n=-\infty}^{\infty} (u(t-t_n) - u(t-t_n-1))$ periyodik işaretinin

üstel Fourier serisi katsayılarını bulun. (G_z)

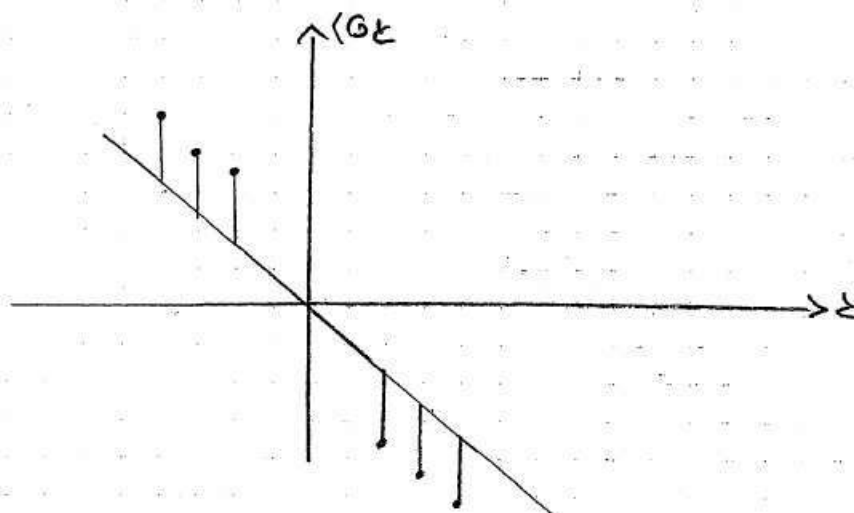
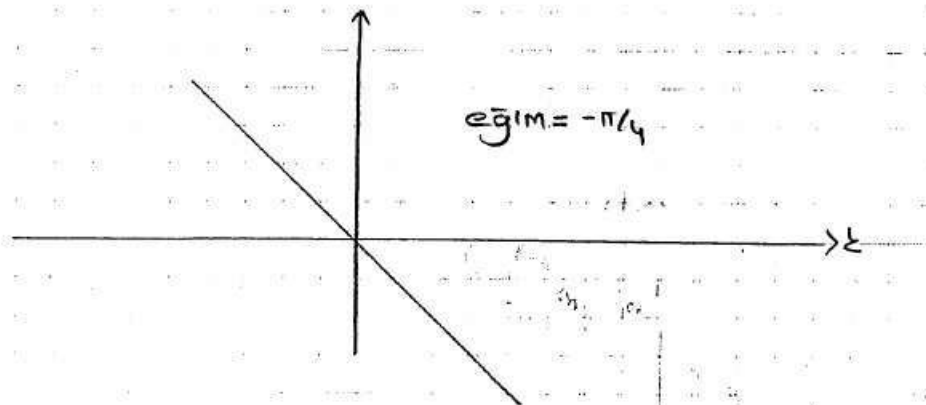
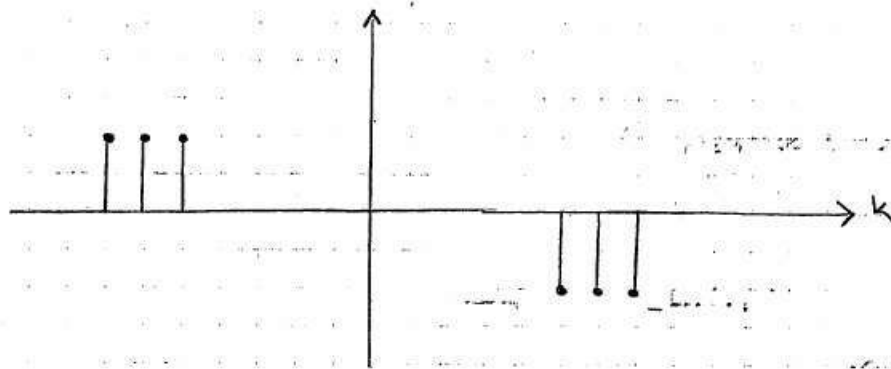
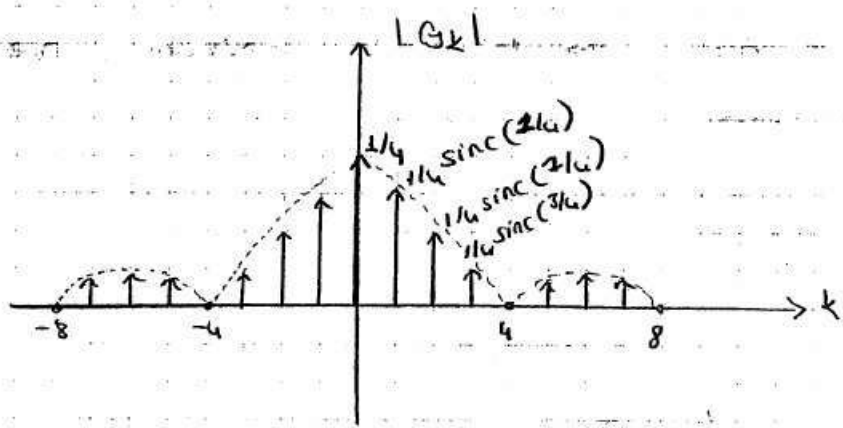
b) Bulduğunuz G_z lerden yararlanarak, genlik ve faz
teyfini t 'nin fonksiyonu olarak ölçekli çiziniz.

a)



$$G(f) = \text{sinc}(cf) e^{-j\omega/2} ; G_z = \frac{1}{T_0} G(t\omega_0)$$

$$\Rightarrow G_k = \frac{1}{4} \text{sinc}(k/4) e^{-j\pi k/4}$$



SORU: Fourier serisi ve fourier dönüşümü arasındaki

ilişiden yararlanarak

a) $g(t) = \sum_{n=-\infty}^{\infty} (u(t+2-6n) - u(t-6n))$ periyodik işaretinin

fourier serisi katsayıları G_k 'leri bulunuz.

b) Bulduğunuz G_k 'lerden yararlanarak genlik ve faz tayfını ω 'nın fonksiyonu olarak örnekli olarak çiziniz.

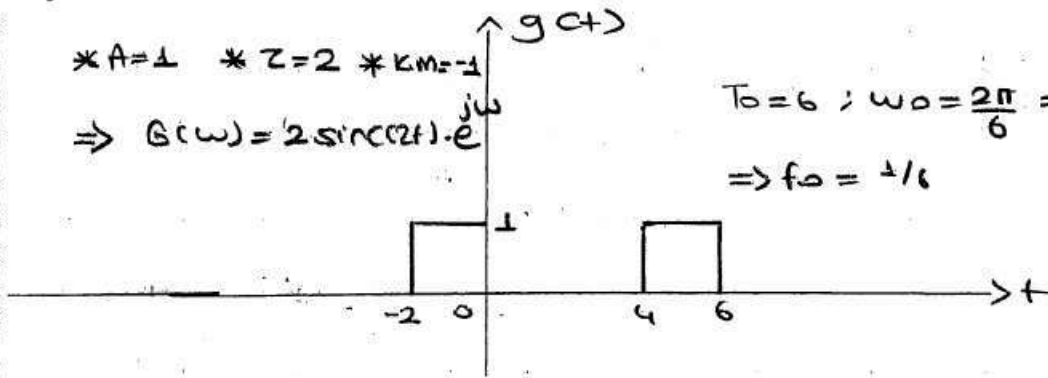
a)

* $A=1$ * $T=2$ * $km=-1$

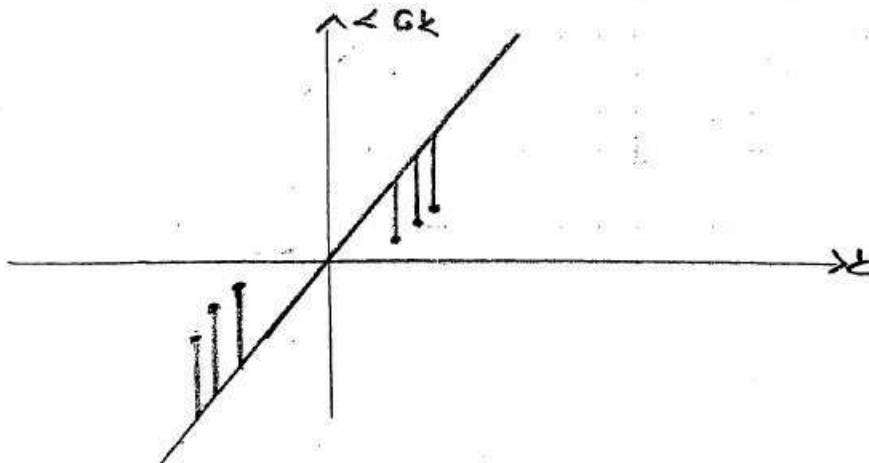
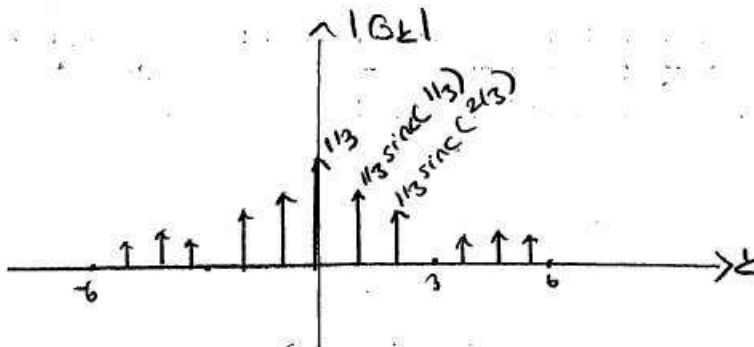
$\Rightarrow G(\omega) = 2 \text{sinc}(\omega T) \cdot e^{j\omega km}$

$T_0=6$; $\omega_0 = \frac{2\pi}{6} = \pi/3$

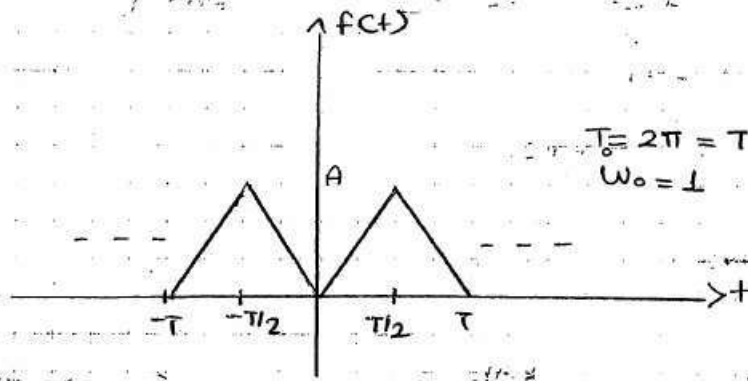
$\Rightarrow f_0 = 1/6$



$\Rightarrow G_k = \frac{1}{3} \text{sinc}(k/3) \cdot e^{jk\pi/3}$ eğim: $\pi/3$



SORU!



* Üstel Fourier serisi katsayılarını bulalım!

$$F_k = \frac{1}{T_0} \int_{T_0} f(t) \cdot e^{-j k \omega_0 t} dt$$

$$\Rightarrow F_k = \frac{1}{2\pi} \times \frac{A}{\pi} \int_{-\pi}^0 e^{-j k t} dt + \frac{1}{2\pi} \times \frac{A}{\pi} \int_0^{\pi} e^{-j k t} dt$$

$$t = u \Rightarrow dt = du$$

$$e^{-j k t} = dv \Rightarrow v = \frac{-1}{j k} e^{-j k t}$$

$$\Rightarrow F_k = \frac{-A}{2\pi^2} \left(\frac{-1}{j k} e^{-j k t} \Big|_{-\pi}^0 + \frac{1}{j k} \int_{-\pi}^0 e^{-j k t} dt \right) + \frac{A}{2\pi^2} \left(\frac{-1}{j k} e^{-j k t} \Big|_0^{\pi} + \frac{1}{j k} \int_0^{\pi} e^{-j k t} dt \right)$$

$$\Rightarrow F_k = \frac{-A}{2\pi^2} \times \left(\frac{-\pi}{j k} \right) e^{j \pi k} - \frac{A}{2\pi^2 j k} \times \frac{(-1)}{j k} e^{-j k t} \Big|_{-\pi}^0 - \frac{A \pi}{2\pi^2 j k} e^{-j \pi k}$$

$$+ \frac{A}{2\pi^2 j k} \times \frac{(-1)}{j k} e^{-j k t} \Big|_0^{\pi}$$

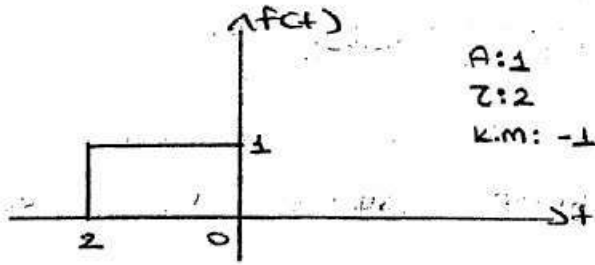
$$\Rightarrow F_k = \frac{A \pi}{2\pi^2} \sin \pi k + \frac{(-A)}{2\pi^2 k^2} (1 - e^{j \pi k}) + \frac{A}{2\pi^2 k^2} (e^{-j \pi k} - 1)$$

$$\Rightarrow F_k = A \sin(\pi k) + \frac{A}{2\pi^2 k^2} (e^{j \pi k} + e^{-j \pi k}) - \frac{A}{\pi^2 k^2}$$

$$\Rightarrow F_k = A \sin(\pi k) + \frac{A}{\pi^2 k^2} \cos \pi k - \frac{A}{\pi^2 k^2}$$

$$\Rightarrow f(t) = \sum_{k=-\infty}^{\infty} \frac{A}{2\pi^2 k^2} (-2 + 2 \cos \pi k + 2 \pi k \sin(\pi k)) \cdot e^{j k t} \quad \text{olur!}$$

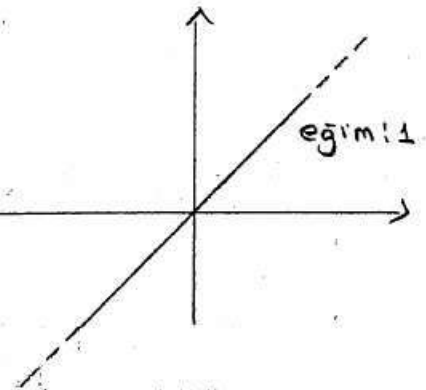
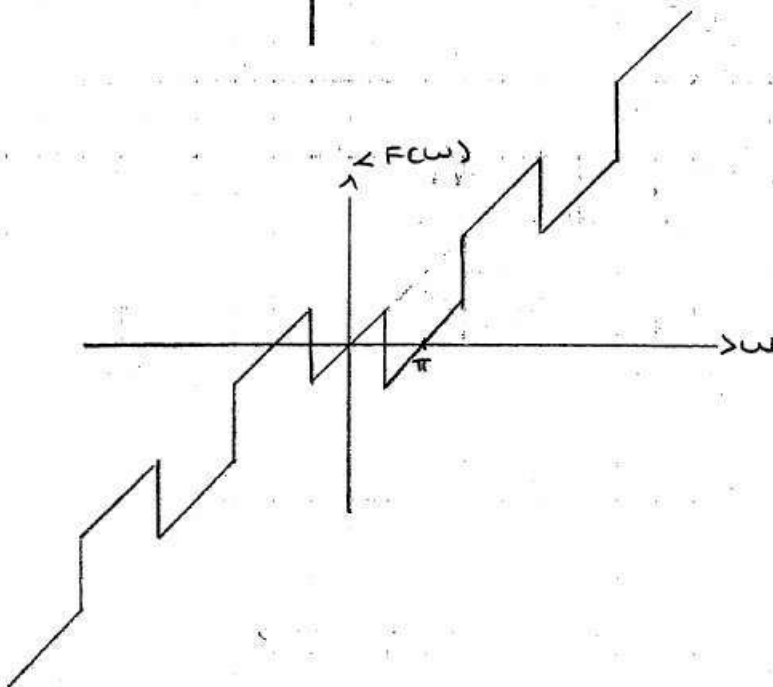
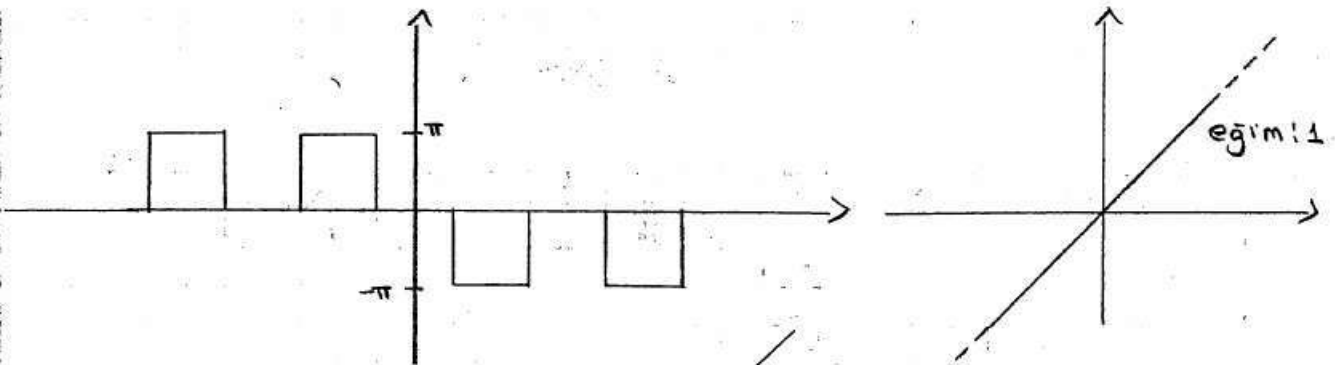
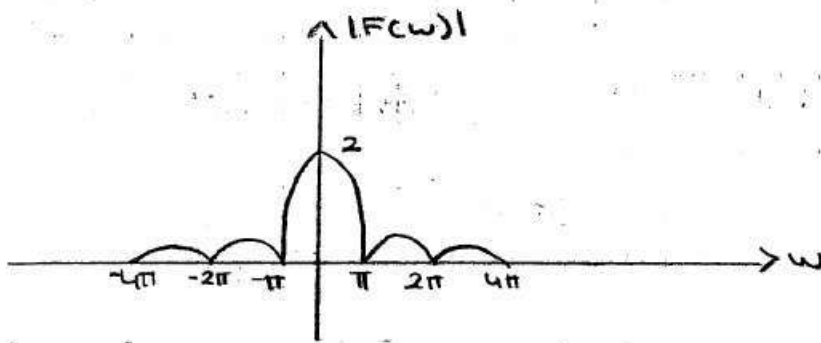
SORU1 $f(t) = u(t+2) - u(t)$ işaretinin fourier dönüşümü $F(\omega)$ -yı bularak iki yanlı genlik ve faz tayfını çiziniz



$A:1$
 $T:2$
 $k.m: -1$

$$\Rightarrow F(\omega) = 2 \operatorname{sinc}(2f) \cdot e^{j\omega}$$

$$\underbrace{2 \operatorname{sinc}\left(\frac{\omega}{\pi}\right)}_{\text{genlik}} e^{j\omega}$$



SORU: $x(t) = 3 \sin(6\pi 10^3 t + \pi/6) - j \cos(12\pi 10^3 t) + 2$, $-\infty < t < \infty$

şinyali veriliyor.

a) $x(t)$ 'nin fourier dönüşümü $X(f)$ 'yi bulunuz.

b) iletkenlik genlik yayfı $|X(f)|$ 'i olaekli olarak aainiz.

c) iletkenlik faz yayfı $\angle X(f)$ 'yi olaekli olarak aainiz.

d) a da verilen $x(t)$ şinyali, frekans transfer fonksiyonu

$$H(f) = \frac{1}{1+j2\pi 10^4 f}$$

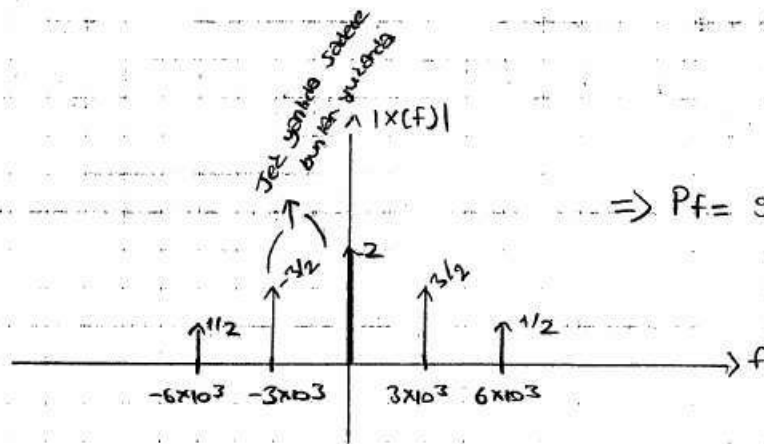
olan bir sistemin giresine veriliyor. Sistemin

ıkışındaki şinyalinin ortalama gıcının gireteki şinyalin ortalama gıcına oranını bulunuz.

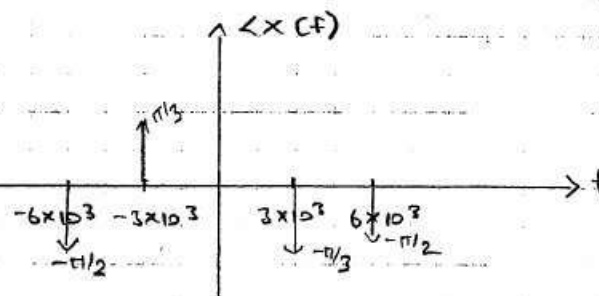
$$\begin{aligned} a) \quad x(t) &= \frac{3}{2j} e^{j(6\pi 10^3 t + \pi/6)} - \frac{3}{2j} e^{-j(6\pi 10^3 t + \pi/6)} \\ &\quad - \frac{j}{2} e^{j(12\pi 10^3 t)} - \frac{j}{2} e^{-j(12\pi 10^3 t)} + 2 \end{aligned}$$

$$\begin{aligned} \Rightarrow X(f) &= \frac{3}{2} (-j) e^{j\pi/6} \delta(f - 3 \times 10^3) + \frac{3}{2} (j) e^{-j\pi/6} \delta(f + 3 \times 10^3) \\ &\quad - \frac{j}{2} \delta(f - 6 \times 10^3) - \frac{j}{2} \delta(f + 6 \times 10^3) + 2 \delta(f) \end{aligned}$$

b)



c)



d) $x(t)$ 'nin üstel fourier serisi katsayılarını bulalım.

$$x(t) = \frac{3}{2j} e^{j\pi/6} x e^{j6\pi 10^3 t} - \frac{3}{2j} e^{-j\pi/6} x e^{-j6\pi 10^3 t} - \frac{j}{2} e^{j12\pi 10^3 t} - \frac{j}{2} e^{-j12\pi 10^3 t} + 2e^{j0t}$$

$f = 3 \times 10^3$ $f = -3 \times 10^3$ $f = 6 \times 10^3$ $f = -6 \times 10^3$ $f = 0$

$$X_{-2} = -j/2, \quad X_{-1} = \frac{3j}{2} e^{-j\pi/6}, \quad X_1 = -\frac{3j}{2} e^{j\pi/6}, \quad X_2 = -j/2, \quad X_0 = 2$$

o hâlde $Y_{-2} = \frac{-j}{2} \times \frac{1}{j-1.2\pi} = \frac{-j}{2} \times 0.256 \times e^{j75.14}$

$$Y_2 = \frac{-j}{2} \times \frac{1}{j+1.2\pi} = \frac{-j}{2} \times 0.256 \times e^{-j75.14} \quad Y_0 = 2$$

$$Y_{-1} = \frac{-3}{2j} \times e^{-j30} \times \frac{1}{j-0.6\pi} = \frac{-1.407}{2j} e^{j32.05}$$

$$Y_1 = \frac{3}{2j} \times e^{j30} \times \frac{1}{j+0.6\pi} = \frac{1.407}{2j} e^{-j32.05}$$

o hâlde $y(t) = \sum_{k=-2}^2 Y_k \cdot e^{j\omega_k t}$

$$\Rightarrow y(t) = \left(0.256 \times e^{j75.14} \times e^{-j12\pi 10^3 t} + 0.256 \times e^{-j75.14} \times e^{j12\pi 10^3 t} \right) / 2j + \left(\frac{-1.407}{2j} e^{j32.05} e^{-j6\pi 10^3 t} + \frac{1.407}{2j} e^{-j32.05} e^{j6\pi 10^3 t} \right) + 2$$

$$\Rightarrow y(t) = 0.256 (-j) \cos(12\pi 10^3 t - 75.14) + 1.407 \sin(6\pi 10^3 t - 32.05) + 2$$

$$\Rightarrow P = \left(\frac{16}{125} \right)^2 \times 2 + \frac{(1.407)^2}{2} + 4 \Rightarrow P = 5 \Rightarrow \text{Oran} = 4.8$$

SORU1 Fourier dönüşümü

$$F(f) = \frac{1}{2j} \delta(f - \sqrt{3}) e^{-j\pi/6} - \frac{1}{2j} \delta(f + \sqrt{3}) e^{j\pi/6} + \delta(f - 5) + \delta(f + 5)$$

olarak bir sinyal veriliyor.

a) Üstel Fourier serisi katsayıları F_k 'leri bulun.

b) İki yarı geniz tayfını ve iki yarı faz tayfını çiziniz.

c) Trigonometrik Fourier serisi katsayıları a_k ve b_k leri bulun.

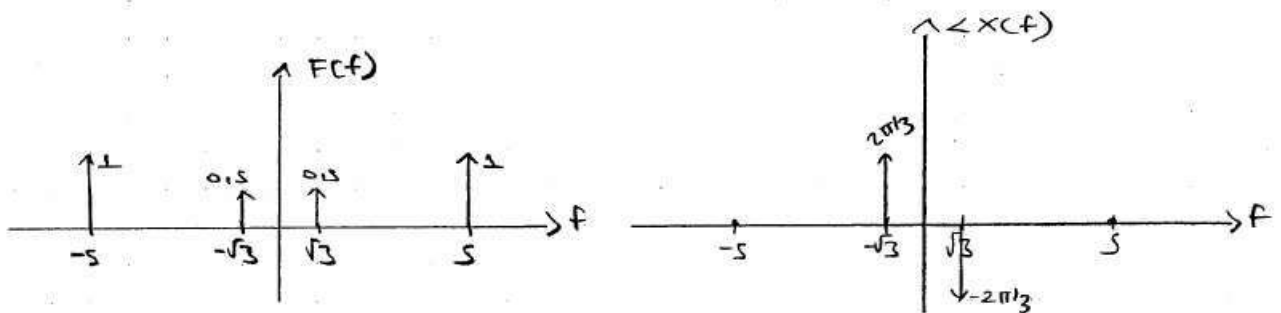
d) Tez yanlı genlik ve faz tayfını çiziniz.

e) $F(f)$ 'nin ters fourier dönüşümü $f(t)$ 'yi bulun.

f) Parseval teoreminden yararlanarak bu sinyalin gücünü zaman domaininde ve frekans domaininde ayrı ayrı hesaplayınız.

a) $F_1 = \frac{1}{2j} e^{-j\pi/6}$, $F_{-1} = \frac{1}{2j} e^{j\pi/6}$, $F_2 = 1$, $F_{-2} = 1$

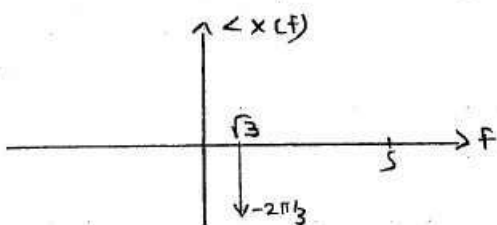
b)



c) $F_z = \frac{az - jbz}{2} \Rightarrow az = 2 \operatorname{Re}\{F_z\}$

$bz = 2jF_z = 2 \operatorname{Im}\{F_z\}$

d)



e) $F(f) = \frac{1}{2j} \delta(f - \sqrt{3}) e^{-j\pi/6} - \frac{1}{2j} \delta(f + \sqrt{3}) e^{j\pi/6}$
 $+ \delta(f - 5) + \delta(f + 5) \Rightarrow$

$\Rightarrow f(t) = \frac{1}{2j} e^{j2\pi\sqrt{3}t} e^{-j\pi/6} - \frac{1}{2j} e^{-j2\pi\sqrt{3}t} e^{j\pi/6} + e^{j2\pi 5t} + e^{-j2\pi 5t}$

$\Rightarrow f(t) = \sin(2\pi\sqrt{3}t - \pi/6) + 2 \cos(10\pi t)$

f) $P_t = \frac{1}{4} + \frac{1}{4} + 1 + 1 = 2.5$ (Fourier series 224. 228. top)

$P_t = \frac{1}{4} + \frac{1}{4} + 1 + 1 = 2.5$ $\int_{-2}^2 (|F(f)|^2) df$

