

## HANDOUT E.1 - EXAMPLES ON SIGNALS AND SYSTEMS

### Signals

A signal is defined as any physical quantity that varies with time, space or any other independent variable or variables. Mathematically, we describe a signal as a function of one or more independent variables.

#### *Examples of signals*

- 1) Current from a current sensor.
- 2) The variation in pressure in the cylinder of an IC engine during operation.
- 3) The variation in the number of shoes sold in a certain shop during the month.
- 4) The variation in temperature at noon on successive days at a holiday resort.

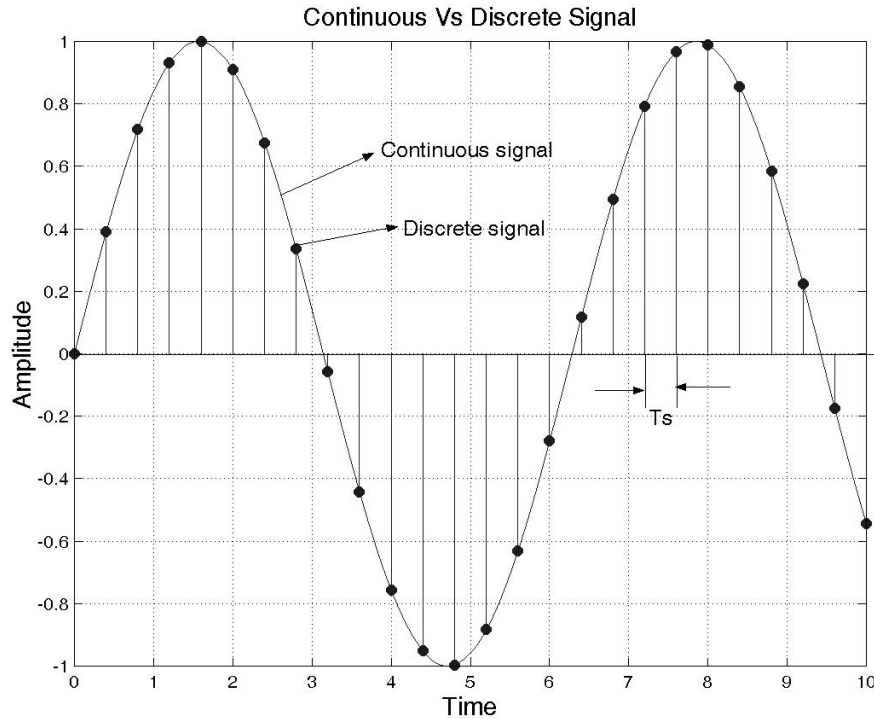
Although all these signals have time as their independent variable, the signals in examples 1 and 2 are basically different from those in examples 3 and 4. In examples 1 and 2 the signals representing the variables are present at all instants of time. These types of signals are known as ***continuous time signals***. In examples 3 and 4 the signals are only available at discrete time intervals. These type of signals are called ***discrete-time signals***. It should be noted that the processes producing the variables in examples 3 and 4 are continuous processes ( the resort temperature has a value at all times as does the number of shoes sold ). However, because the measurements are taken at discrete intervals, they give rise to discrete-time signals.

The discrete-time signals arise due to the *sampling* of a continuous signal. Selecting values of an analog signal at discrete-time instants is called sampling. All measuring instruments that take measurements at a regular interval of time provide discrete-time signals.

For example, counting the number of cars using a street every hour or recording the value of gold every day results in discrete-time signals.

The continuous time signals can also be defined as those signals that can take any value at any given instant of time. Whereas the discrete-time signals are defined only for discrete instants of time. That is the time variable of these signals can take only certain values.

The difference between a continuous and a discrete-time signal is explained further with the help of the following figure.



The continuous signal in the above figure is represented by

$$x(t) = \sin t.$$

To obtain the discrete-time signal, values of the continuous time signals are taken at equally spaced time intervals of 0.4 seconds. In other words, the continuous time signal is sampled at the rate of  $\frac{1}{0.4}$  samples per second or 2.5 samples per second. So the discrete time signal is represented as

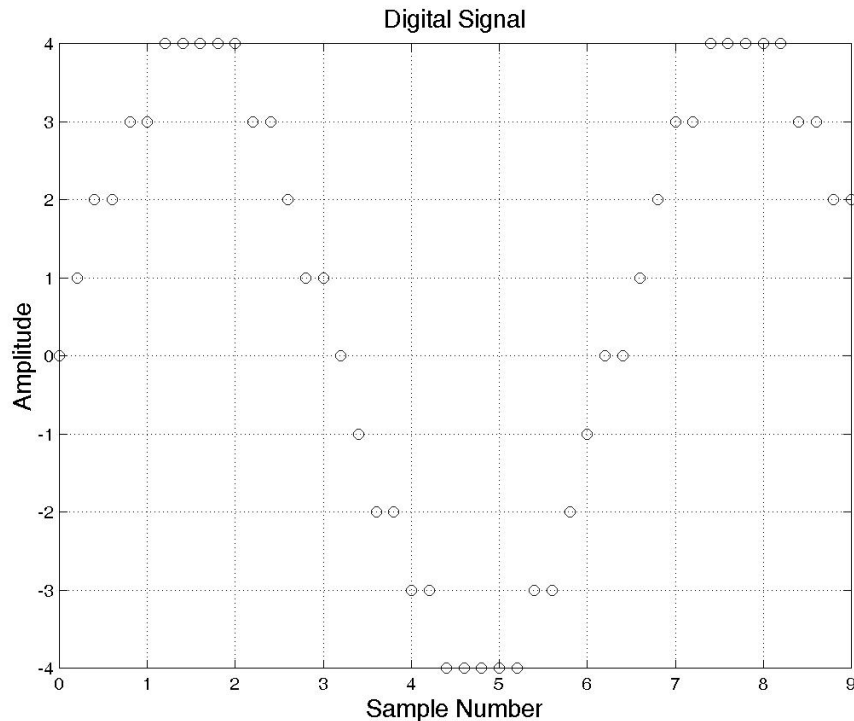
$$x(n) = \sin(0.4n).$$

Therefore in general, if the continuous time signal is represented as

$x(t) = f(t)$ , then the discrete-time signal, which is obtained by sampling the continuous time signal every  $T$  seconds is represented as

$$x(n) = f(nT).$$

Quantization of the discrete signals gives rise to **digital signals or discrete valued signals**. A quantized signal assumes only discrete amplitude values. In other words, in these signals both the amplitude and time variable can take only certain values. A digital signal is shown in the figure given below.

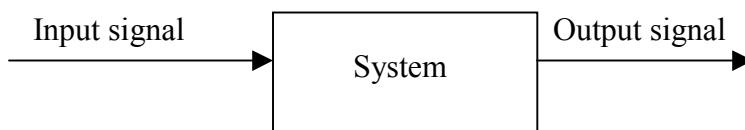


## Systems

A system may be defined as a physical device that performs an operation on a signal. It can also be regarded as a process that transforms one signal into another. The input to a system and the output from a system are signals.

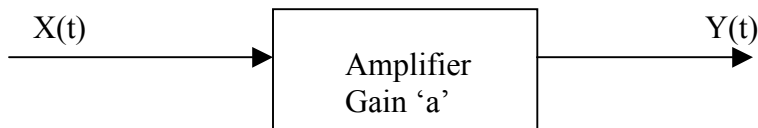
A system can also be defined as a group of physical components assembled to perform a specific function. It may be electrical, mechanical, hydraulic, pneumatic, thermal or a combination of any of these systems.

In general a system may be represented as



All existing systems change with time and when the rates of change are significant, the systems are referred to as dynamic systems. A car riding on a road can be considered to be a dynamic system (especially on a crooked or a bumpy road). The input variables could be the throttle position, position of the steering wheel or the road conditions like the slope and roughness. The choice of the output variables is arbitrary and is determined by the objective of the analysis. The position, velocity or acceleration of the car or the average fuel rate or the engine temperature can be selected as the outputs.

Another example of a system is an amplifier, which modifies the amplitude of the input signal by a gain factor at every time instant. This system is shown below.



In general the relationship between the input signal to the system ' $X(t)$ ' and the output signal from the system, ' $Y(t)$ ' is given by

$Y(t) = aX(t)$ , where ' $a$ ' is a constant.

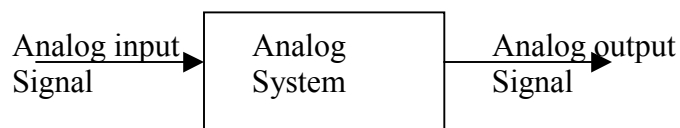
### System modeling and types of models

The main objective of system analysis is to predict relevant performance characteristics of a system operating under specified environmental conditions. To understand and control complex systems, one must therefore establish an adequate mathematical model of the system. The term **model** means a set of differential equations that describe the dynamic behavior of the system.

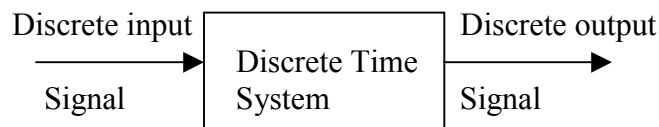
System models can be classified into various types. Some of the types are considered below.

*Continuous time systems, discrete time systems and digital systems*

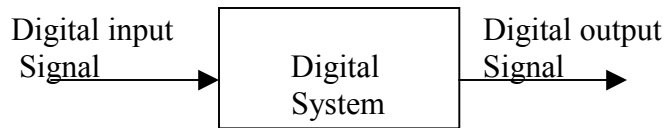
Systems where the input and the output signals are continuous in nature are *continuous systems*. Example of a continuous time system or an analog system is a sensor. Continuous systems can be expressed in the form of differential equations.



Systems where both the input and the output signals are discrete in nature are called discrete-time systems. These systems are expressed in the form of time-difference equations.



If both the input and output of a system are digital in nature, then the system is called a digital system. Example of a digital system is a computer.



### *Linear and Non-linear systems*

The concept of linear system can be illustrated by an example. For example, one is listening to some instrumental music from a tape via an amplifier. One instrument plays the tune, this instrument stops and another instrument plays the same tune, then both instruments play the tune together. When the instruments play together, one would expect the sound produced to be the combination of the sounds of the individual instruments and if the system is linear this will be the case. But we all know that this is not the case. Hence the basis of linear system is that if the inputs are superimposed then the responses to these individual inputs are also superimposed. In other words

If an arbitrary input  $x_1(t)$  produces the output  $y_1(t)$  and an arbitrary input  $x_2(t)$  produces an output  $y_2(t)$ , then if the system is linear input  $(x_1(t)+x_2(t))$  will produce an output  $(y_1(t)+y_2(t))$ . This principle is called the principle of superposition. All linear systems satisfy the principle of superposition. One example of a linear system is the simplified model of the suspension system of an automobile (spring-mass-damper system). Example of a non-linear system is a pendulum of a clock. Non-linear systems do not satisfy the principle of superposition.

### *Time domain and frequency domain models*

For any simple system, the mathematical model is obtained by applying the laws of physics. The result is a differential equation. This equation is nothing but the time domain model of the physical system. By taking the Laplace transform of the differential equation, we get the frequency domain model of the same physical system. So time domain and frequency domain models represent the same physical system in two different domains.

For example, consider the suspension system of an automobile, which can be simplified into a mass-spring-damper system. The governing differential equation of motion for the system is given by

$m\ddot{x} + c\dot{x} + kx = f$ . ' $f$ ' is the input to the system. This equation represents the time domain model of the system under consideration.

Taking the Laplace transform of the above equation, we get

$$ms^2 X(s) + csX(x) + kX(s) = F(s),$$

$$\Rightarrow \frac{X(s)}{F(s)} = G(s) = \frac{1}{ms^2 + cs + k}.$$

G(s) represents the frequency domain model of the same physical system considered.