

EEM322- Lecture4

Block diagram representation of systems

Signal-flow graphs

Block diagram Reduction

Signal-flow graphs Reduction

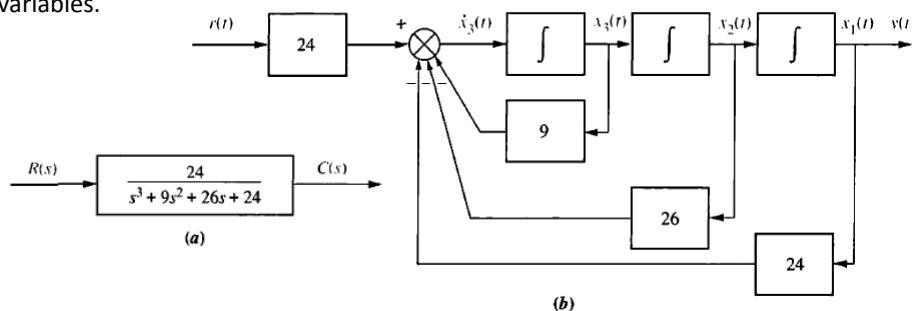
Mason's Rule

Multiple subsystems are represented in two ways:

As block diagrams (block diagrams are usually used for frequency-domain analysis)

As signal-flow graphs, (signal-flow graphs are usually used for state-space analysis).

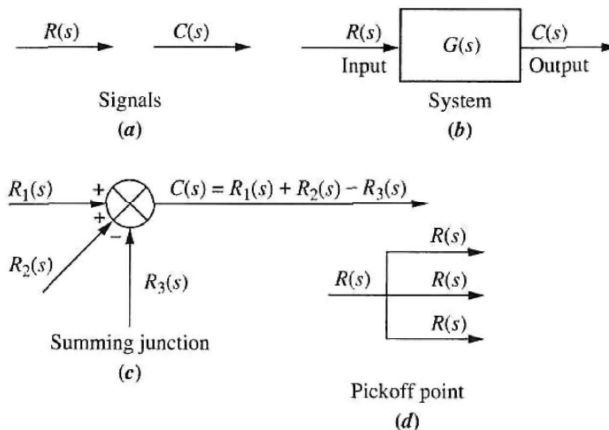
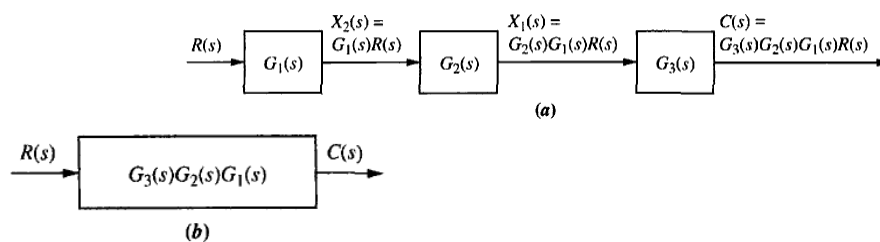
Signal-flow graphs represent transfer functions as lines, and signals as small circular nodes. Summing is implicit. To show why it is convenient to use signal-flow graphs for state-space analysis and design, consider Figure . A graphical representation of a system's transfer function is as simple as Figure (a). However, a graphical representation of a system in state space requires representation of each state variable, as in Figure (b). In that example, a single-block transfer function requires seven blocks and a summing junction to show the state variables explicitly. Thus, signal-flow graphs have advantages over block diagrams, such as Figure (b): They can be drawn more quickly, they are more compact, and they emphasize the state variables.



Block Diagrams:

A subsystem is represented as a **block with an input, an output**, and a transfer function. **Many systems are composed of multiple subsystems.**

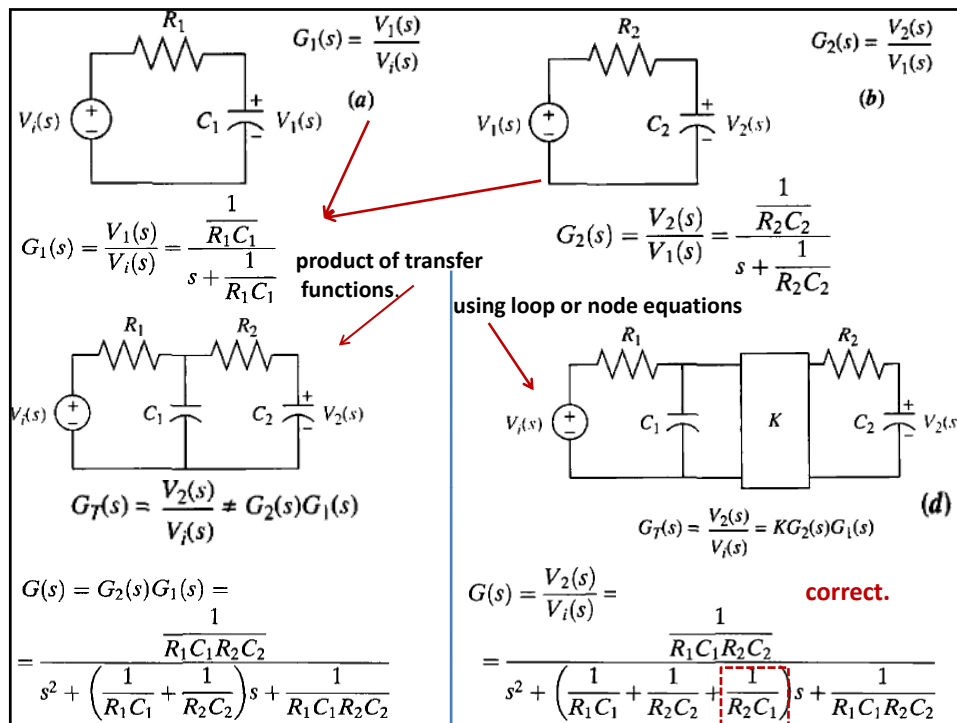
When multiple subsystems are interconnected, a few more schematic elements must be added to the block diagram. These new elements are *summing junctions* and *pickoff points*. All component parts of a block diagram for a linear, time-invariant system are shown in Figure

**Cascade Form:** the product of the subsystems' transfer functions

Equation (b) was derived under the assumption that interconnected subsystems **do not load adjacent subsystems**. That is, a subsystem's output remains the same whether or not the subsequent subsystem is connected. **If there is a change in the output, the subsequent subsystem loads the previous subsystem**, and the equivalent transfer function is not the product of the individual transfer functions.

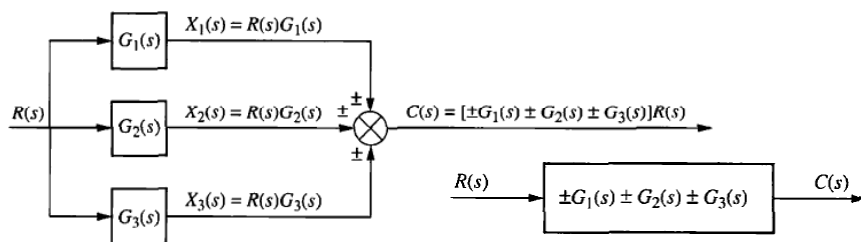
prevent loading

One way to prevent loading is to use an amplifier between the two networks, as shown in Figure (d). **The amplifier has a high-impedance input, so that it does not load the previous network.** At the same time it has a low-impedance output, so that it looks like a pure voltage source to the subsequent network. With the amplifier included, the equivalent transfer function is the product of the transfer functions and the gain, K , of the amplifier.

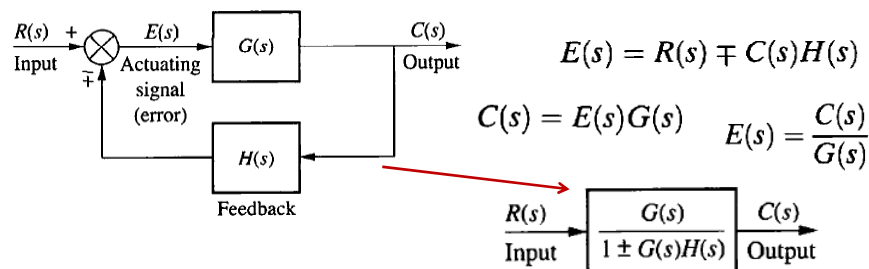


Parallel Form:

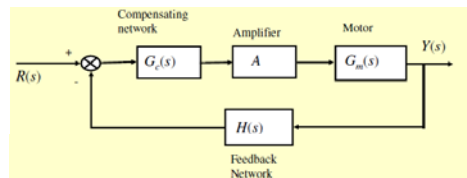
By writing the output of each subsystem, we can find the equivalent transfer function. Parallel subsystems have a common input and an output formed by the algebraic sum of the outputs from all of the subsystems.



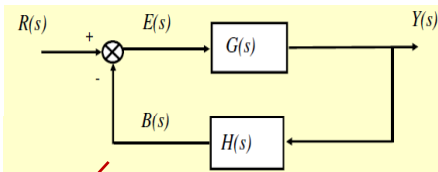
Feedback Form



A feedback system:
negative feedback



Generalized Block Diagram
- (single-input single-output systems)



$R(s)$ = reference input

$Y(s)$ = controlled output

$E(s)$ = actuating error

$B(s)$ = feedback variable

$G(s)$ = forward transfer function (FTF)

$H(s)$ = feedback transfer function

$G(s)H(s)$ = loop transfer function or open loop transfer function (OLTF)

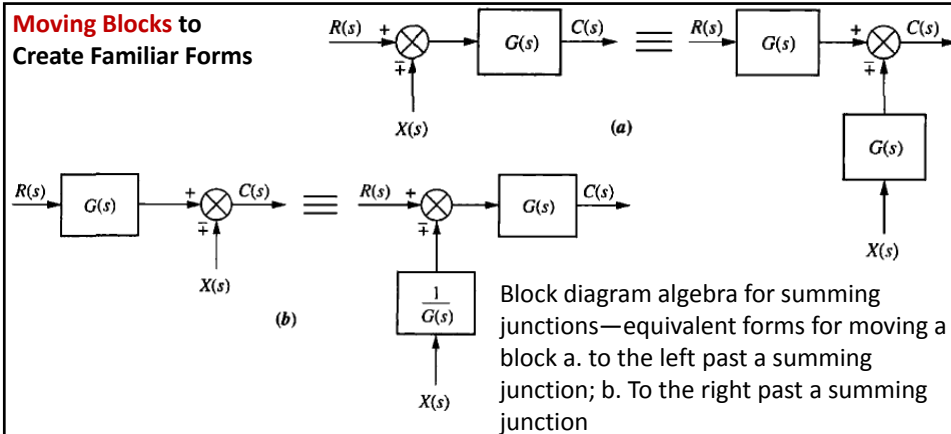
$\frac{Y(s)}{R(s)}$ = overall transfer function or closed-loop transfer function (CLTF)

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{FTF}{1 + OLTF}$$

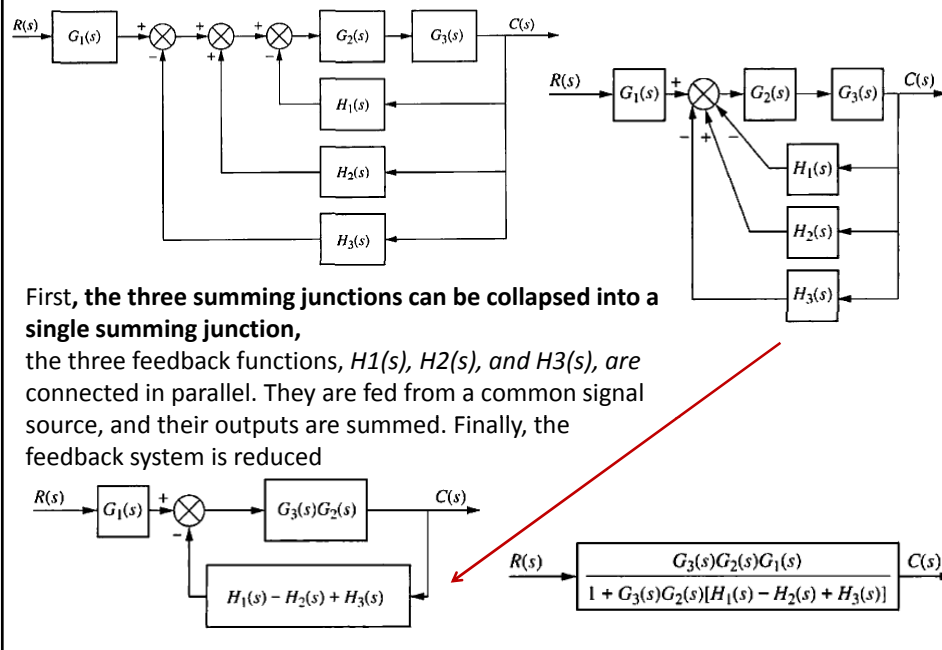
$$1 + G(s)H(s) = 0$$

characteristic equation

**Moving Blocks to
Create Familiar Forms**



PROBLEM: Reduce the block diagram shown in Figure



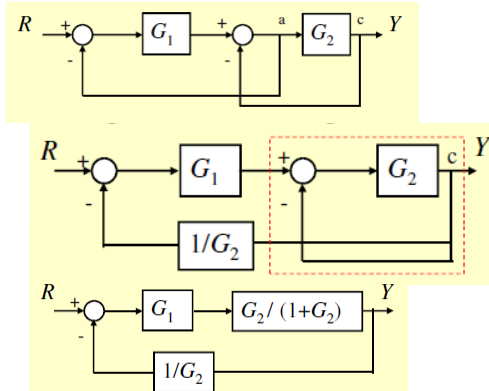
Original Block Diagram	Manipulation	Modified Block Diagram
	Cascaded elements	
	Addition or subtraction (eliminating auxiliary forward path)	
	Shifting of pickoff point ahead of block	
	Shifting of pickoff point behind block	
	Shifting summing point ahead of block	
	Shifting summing point behind block	
	Removing H from feedback path	
	Eliminating feedback path	

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Exercise: Consider the block diagram given below.

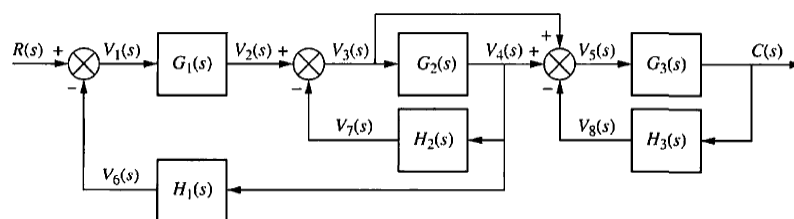
Move take-off point a to c. We get

Simplifying the outlined feedback loop, we get:

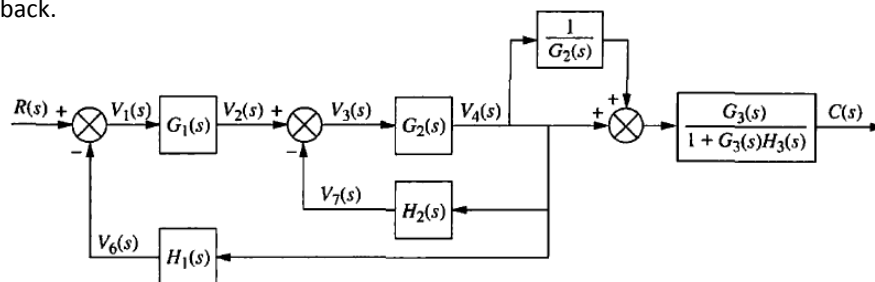


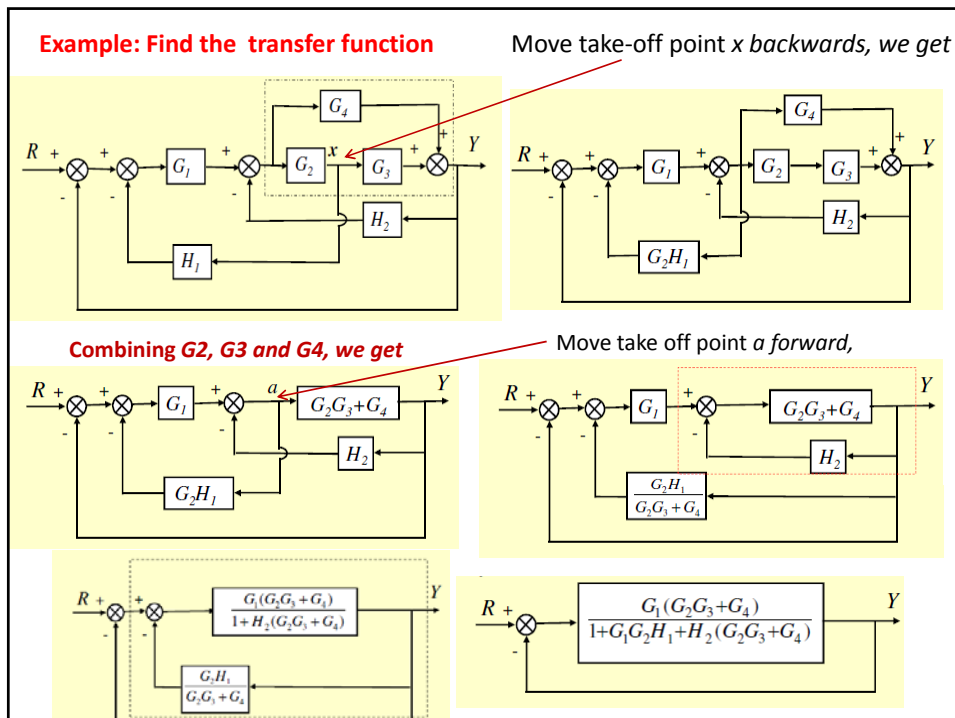
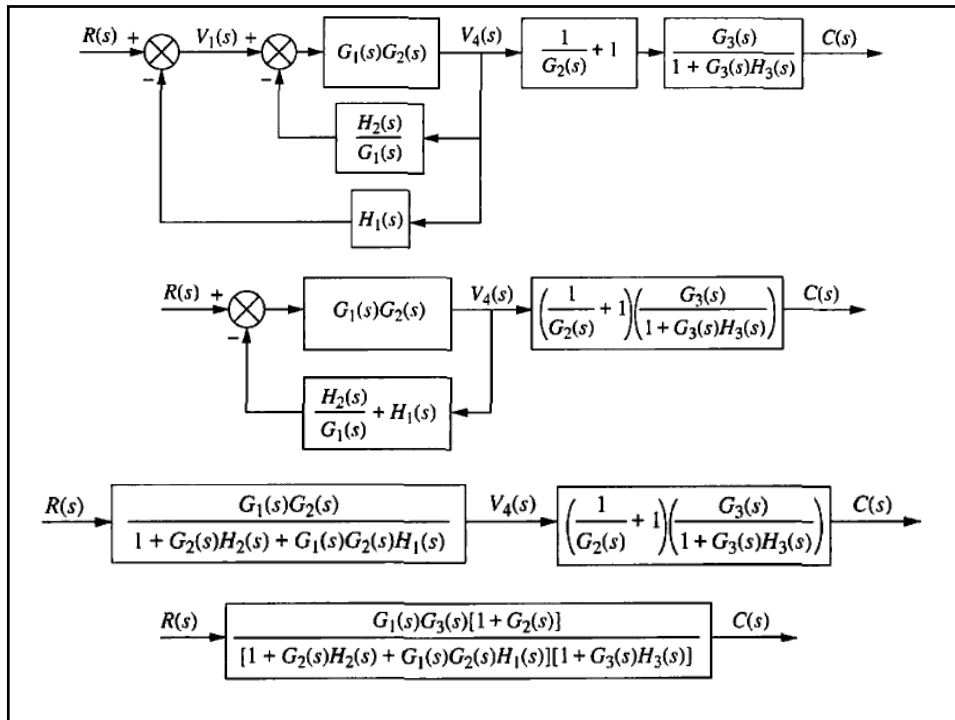
$$\text{Hence, } \frac{Y}{R} = \frac{G_1 \cdot \frac{G_2}{1+G_2}}{1 + G_1 \cdot \frac{G_2}{1+G_2} \cdot \frac{1}{G_2}} = \frac{G_1 G_2}{1 + G_1 + G_2}$$

Block Diagram Reduction by Moving Blocks

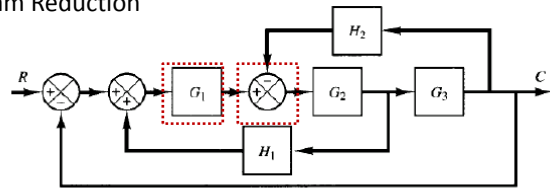


First, **move $G_2(s)$ to the left past the pickoff point to create parallel subsystems**, and reduce the feedback system consisting of $G_3(s)$ and $H_3(s)$. This result is shown in Figure. Second, **reduce the parallel pair consisting of $VG_2(s)$ and unity, and push $G_1(s)$ to the right past the summing junction**, creating parallel subsystems in the feedback.

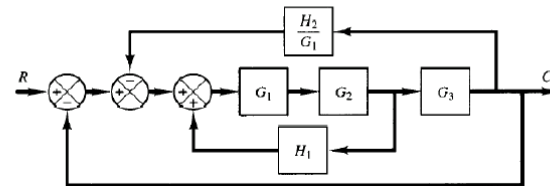




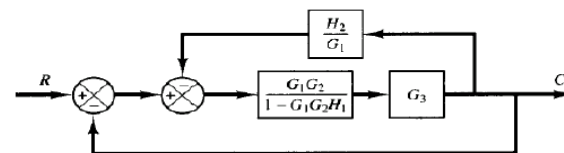
Block diagram Reduction



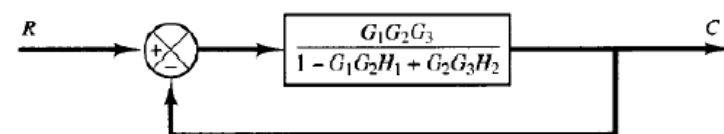
Move the summing point of the negative feedback loop containing H_2 outside the positive feedback loop containing H_1 .



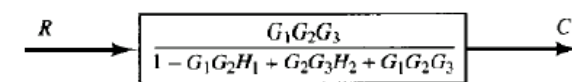
Eliminate the positive feedback loop containing H_1 .



Eliminate the feedback loop containing H_2/G_1



Eliminate the feedback loop

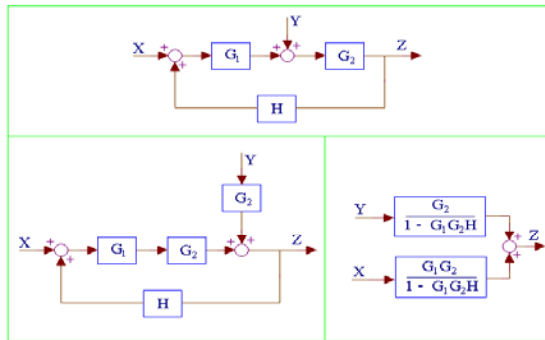


Closed-loop transfer function:

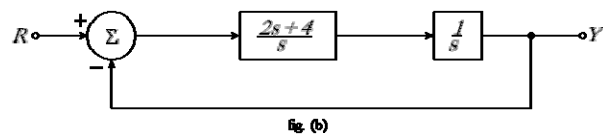
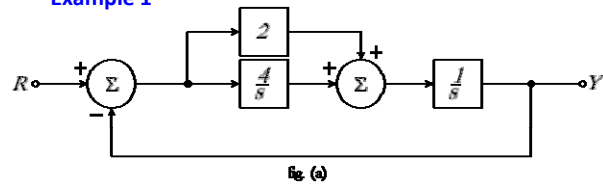
$$\frac{C(s)}{R(s)} = \frac{\text{Product of the transfer functions of the feedforward path}}{1 - \sum \text{product of the transfer functions around each loop}}$$

Note: positive feedback loop yields a negative term in the denominator

Equivalent: 2



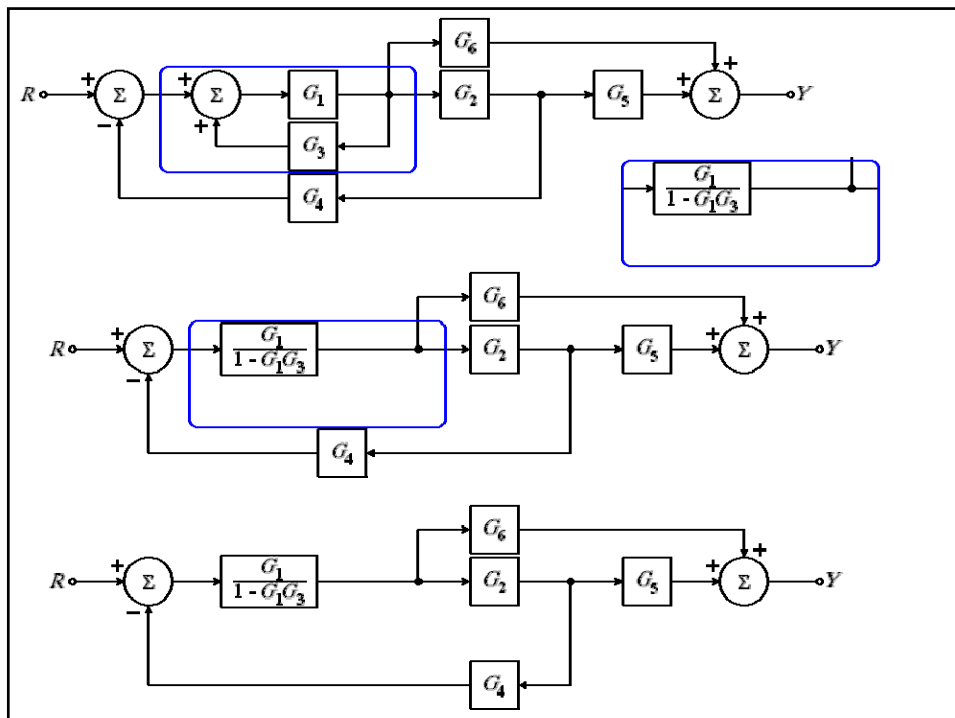
Example 1

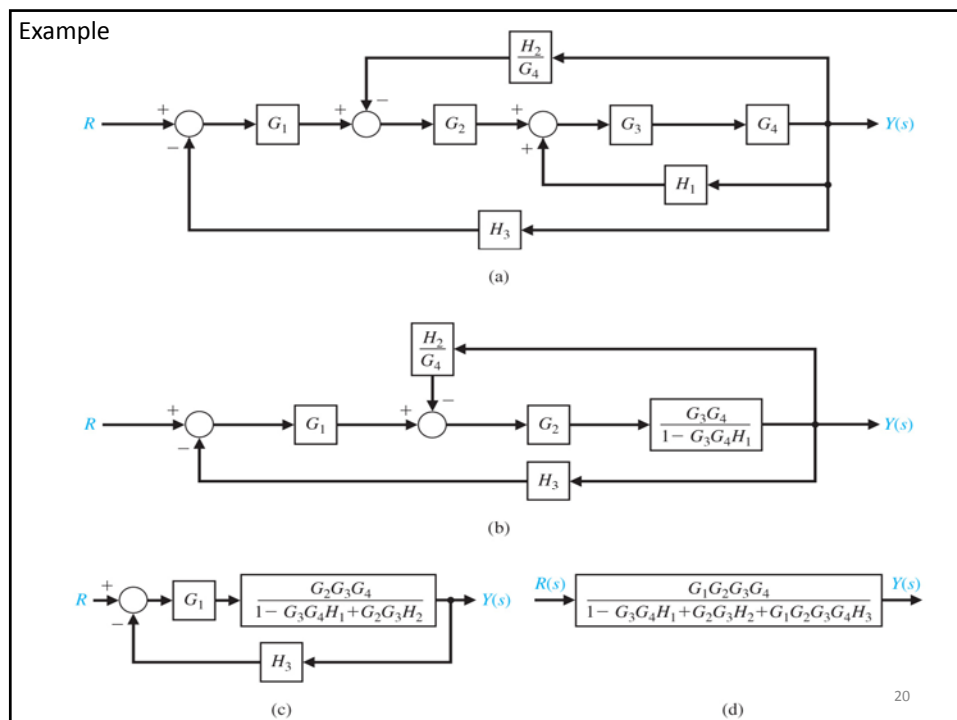
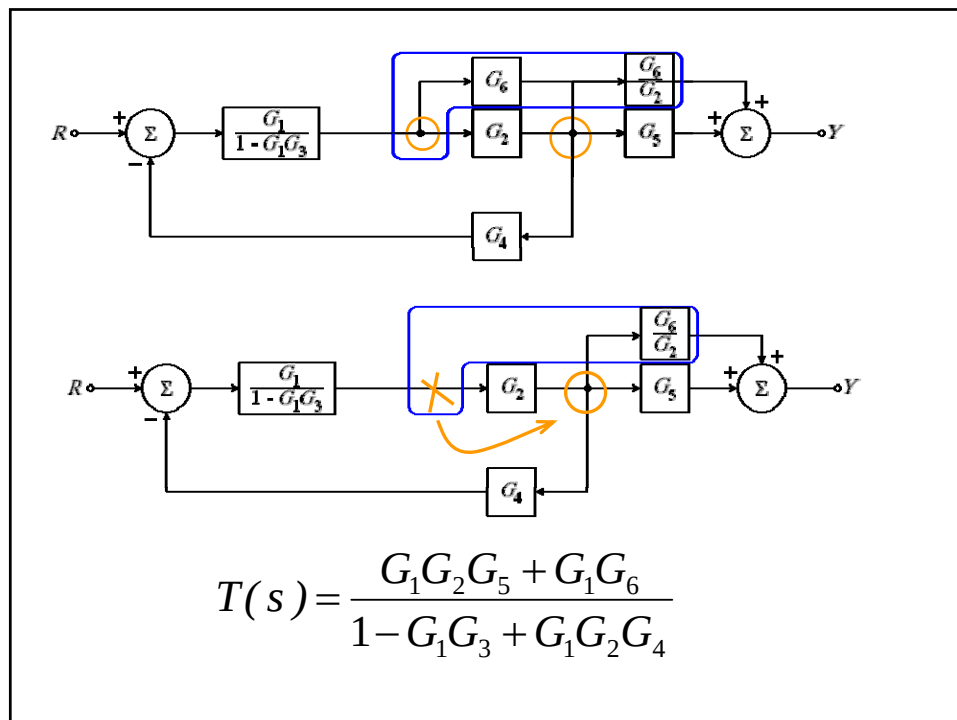


$$T(s) = \frac{Y(s)}{R(s)}$$

$$T(s) = \frac{\frac{2s+4}{s^2}}{1 + \frac{2s+4}{s^2}}$$

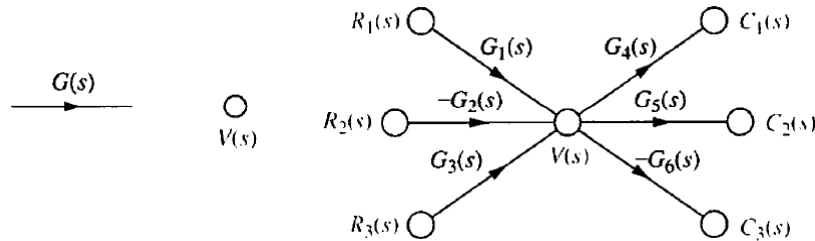
$$T(s) = \frac{2s+4}{s^2 + 2s + 4}$$





Signal-Flow Graphs

Signal-flow graphs are an alternative to block diagrams. a signal-flow graph consists only of *branches*, which represent systems, and *nodes*, which represent signals. These elements are shown in Figure



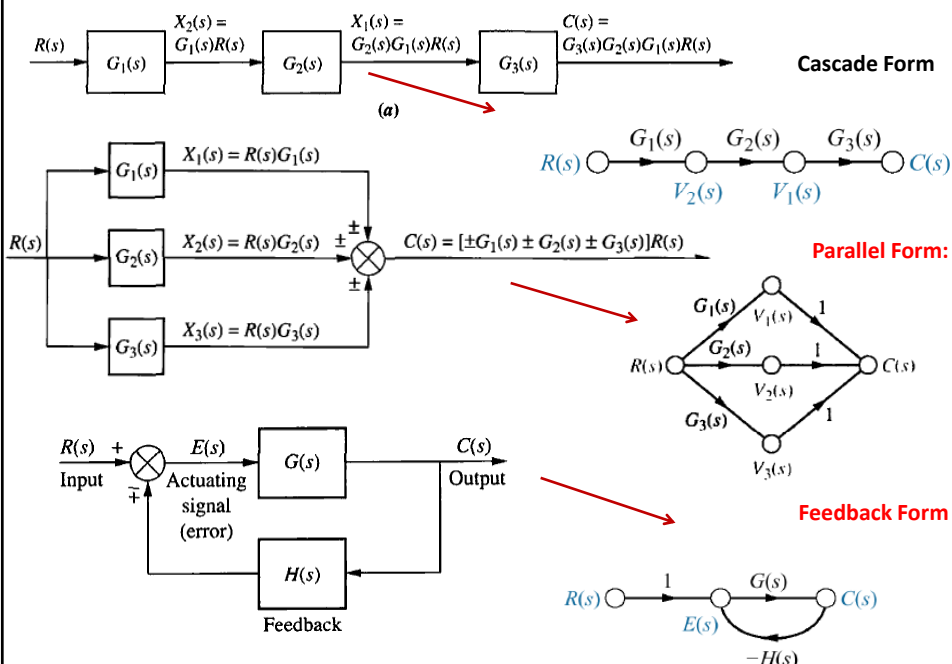
For example, the signal

$$V(s) = R_1(s)G_1(s) - R_2(s)G_2(s) + R_3(s)G_3(s).$$

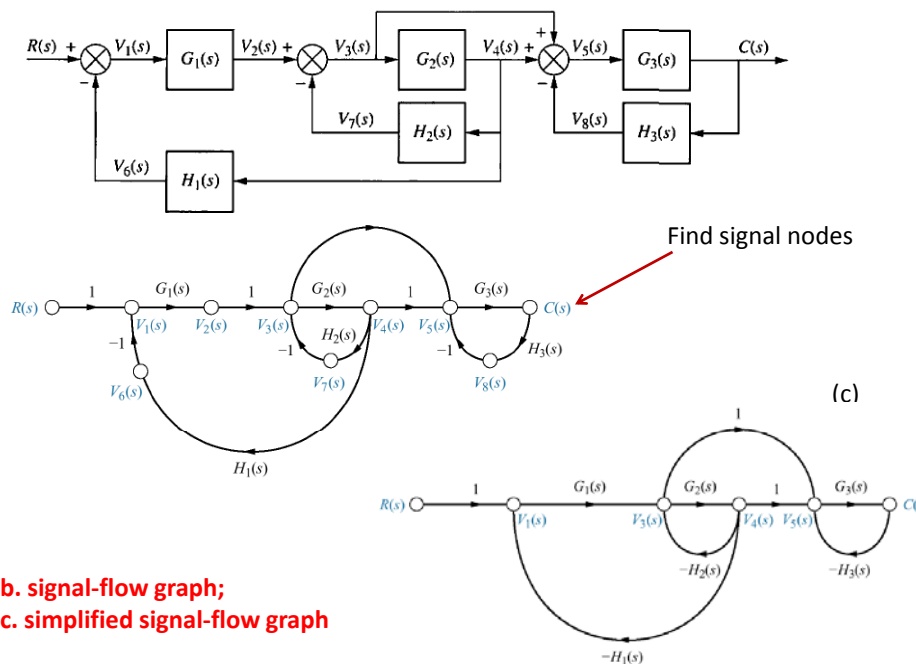
$$C_3(s) = -V(s)G_6(s) = -R_1(s)G_1(s)G_6(s) + R_2(s)G_2(s)G_6(s) - R_3(s)G_3(s)G_6(s)$$

$$G_6(s) + R_2(s)G_2(s)G_6(s) - R_3(s)G_3(s)G_6(s)$$

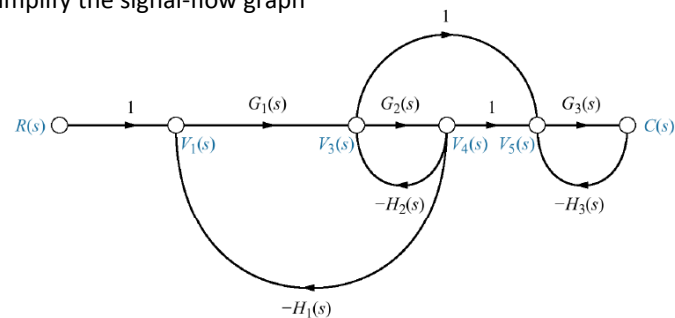
Converting Common Block Diagrams to Signal-Flow Graphs



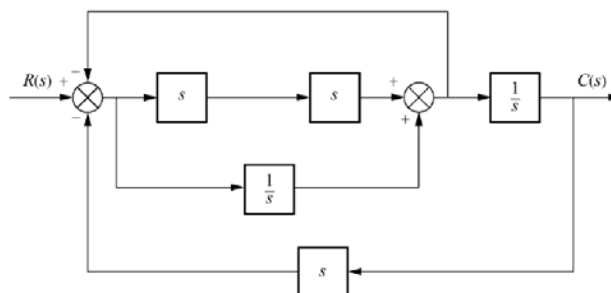
Converting a Block Diagram to a Signal-Flow Graph



Homework: Simplify the signal-flow graph



Convert the block diagram of figure below to a signal-flow graph



MASON'S GAIN RULE for reduction

$$M = \frac{\sum_j M_j \Delta_j}{\Delta}$$

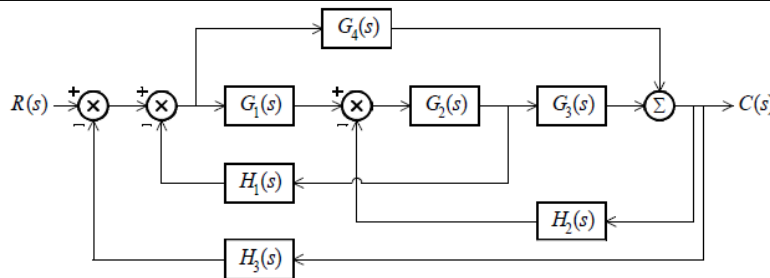
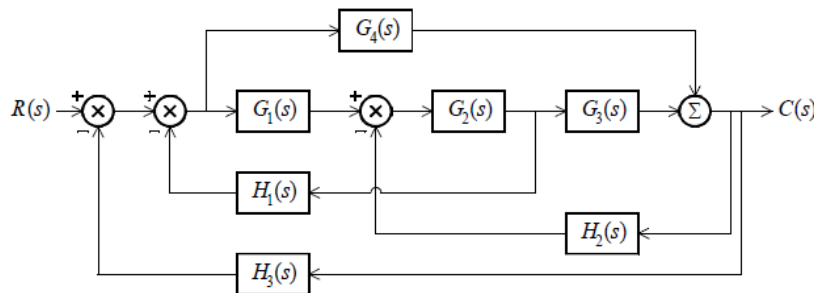
M = transfer function or gain of the system

M_j = gain of one forward path

j = an integer representing the forward paths in the system

$\Delta_j = 1 -$ the loops remaining after removing path j . If none remain, then $\Delta_j = 1$.

$\Delta = 1 - \Sigma$ loop gains $+ \Sigma$ nontouching loop gains taken two at a time $- \Sigma$ nontouching loop gains taken three at a time $+ \Sigma$ nontouching loop gains taken four at a time $- \dots$



Find the forward paths and their gains:

A forward path is a path from $R(s)$ to $C(s)$ that does not cross the same point more than once. There are two forward paths in this example, so we have a $j = 1$ and a $j = 2$. The two paths are:

$$M_1 = G_1 G_2 G_3 \quad \text{and} \quad M_2 = G_4$$

Find the loops and their gains:

A loop is a closed path that can be negotiated without crossing the same point more than once. There are five loops in this example:

$$\text{Loop 1} = -G_1 G_2 H_1$$

$$\text{Loop 2} = -G_2 G_3 H_2$$

$$\text{Loop 3} = -G_1 G_2 G_3 H_3$$

$$\text{Loop 4} = -G_4 H_3$$

$$\text{Loop 5} = G_4 H_2 G_2 H_1$$

Find the Δ_j s:

If we eliminate the path $M_1 = G_1 G_2 G_3$ from the system, no complete loops remain so:

$$\Delta_1 = 1$$

If we eliminate the path $M_2 = G_4$ from the system, no complete loops remain so:

$$\Delta_2 = 1$$

Since there are no nontouching loop pairs in this example, Δ will just be one minus the sum of the loop gains:

$$\Delta = 1 - [(-G_1 G_2 H_1) + (-G_2 G_3 H_2) + (-G_1 G_2 G_3 H_3) + (-G_4 H_3) + (G_4 H_2 G_2 H_1)]$$

$$\text{Mason's Rule: } M = \frac{\sum_j M_j \Delta_j}{\Delta}$$

$$\frac{C(s)}{R(s)} = M = \frac{G_1 G_2 G_3 + G_4}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 H_3 + G_4 H_3 - G_4 H_2 G_2 H_1}$$

Path : – A continuous connection of branches in the same direction

Loop: – A closed path in which no node is traveled through twice

Forward Path: – A path that connects a source node to a sink node in which no node is traversed more than once

Loop Gain: – The product of all the transfer functions in the loop

Nontouching: – Two loops are nontouching if they have no nodes in common

– A loop and a path are nontouching if they have no nodes in common

Transfer Function

- The transfer function from a source to a sink node

p is the number of forward paths

- M_k is the path gain for the kth forward path

$$T = \frac{1}{\Delta} \sum_{k=1}^p M_k \Delta_k$$

$$\Delta = 1 - \sum \text{individual loops} + \sum \text{product of nontouching loops taken two at a time} - \sum \text{product of nontouching loops taken three at a time} + \dots$$

Mason's Rule:

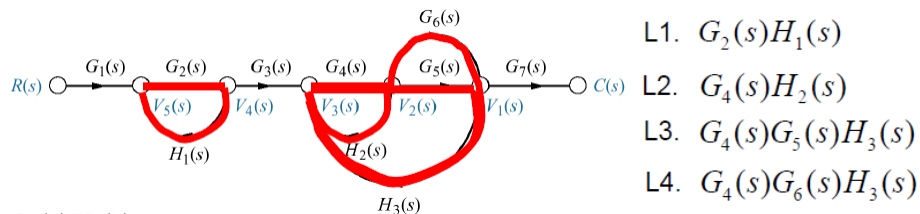
A technique to reduce a signal-flow graph to a single transfer function. Requires the application of one formula. The transfer function, $C(s)/R(s)$, of a system represented by a signal-flow graph is

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

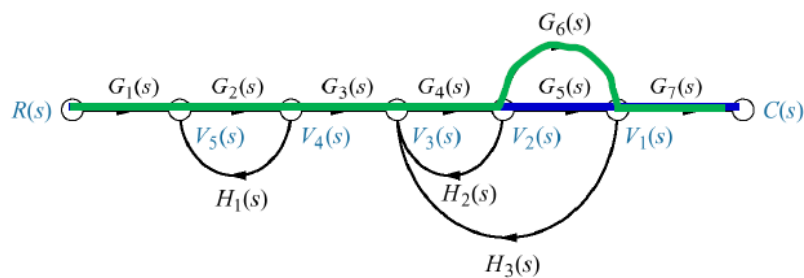
- k = number of forward path
- T_k = the k th forward path gain
- $\Delta = 1 - (\sum \text{loop gains}) + (\sum \text{non-touching loop gains taken two at a time}) - (\sum \text{non-touching loop gains taken three at a time})$.
- $\Delta_k = 1 - (\text{loop-gain which does not touch the forward path})$

Mason's Rule – definitions

Loop gain – the product of branch gains found by traversing a path that starts at a node and ends at the same node

**Mason's Rule – definitions**

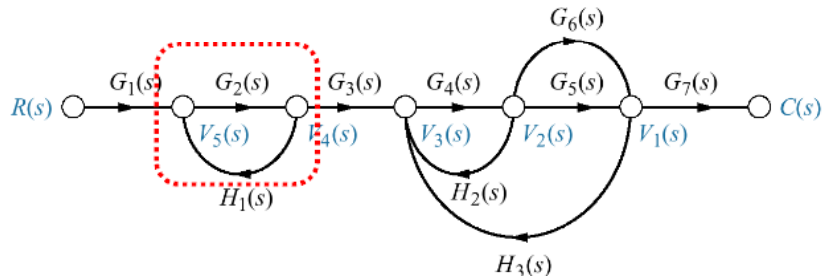
Forward path – the product of gains found by traversing a path from the input node to the output node of the signal-flow graph



Mason's Rule – definitions

Non-touching loop – Loops that do not have any nodes in common.

Non-touching loop gain – The product of loop gains from non-touching loops taken two, three, four, or more at a time

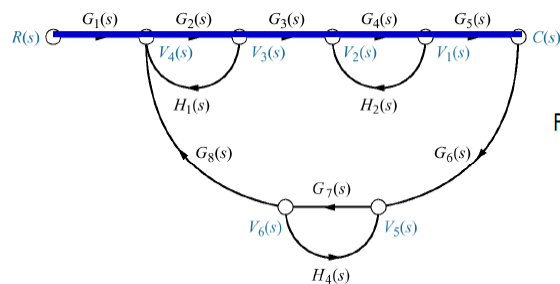


$$\text{NT1. } [G_2(s)H_1(s)][G_4(s)H_2(s)]$$

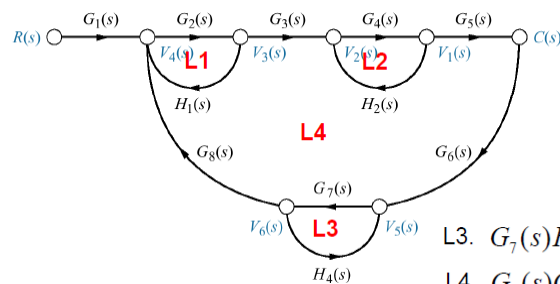
$$\text{NT2. } [G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$$

$$\text{NT3. } [G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$$

Example : Find the transfer function, $C(s)/R(s)$, for the signal-flow graph in figure below



$$\text{F1. } G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$$



$$\text{L1. } G_2(s)H_1(s)$$

$$\text{L2. } G_4(s)H_2(s)$$

$$\text{L3. } G_7(s)H_4(s)$$

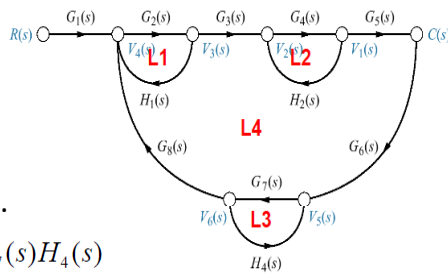
$$\text{L4. } G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$$

Third, identify the **non-touching loops taken two at a time**.

L1 and L2: $G_2(s)H_1(s)G_4(s)H_2(s)$

L1 and L3: $G_2(s)H_1(s)G_7(s)H_4(s)$

L2 and L3: $G_4(s)H_2(s)G_7(s)H_4(s)$



non-touching loops taken three at a time.

L1, L2, L3: $G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$

Now, form the Δ and Δk .

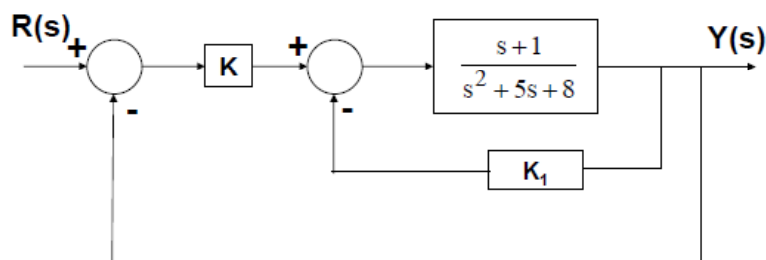
$$\begin{aligned}\Delta = & 1 - [G_2(s)H_1(s) + G_4(s)H_2(s) \\ & + G_7(s)H_4(s) + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)] \\ & + [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) \\ & + G_4(s)H_2(s)G_7(s)H_4(s)] \\ & - [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]\end{aligned}$$

Δk is formed by $[1 - \text{loop does not touch forward path}]$

$$\Delta_1 = 1 - G_7(s)H_4(s)$$

APPLICATION OF THE MASON FORMULA

$$H(s) = \frac{\sum \text{Forward Paths}}{1 - \sum \text{Closed Loops}}$$



$$H(s) = \frac{Y(s)}{R(s)} = \frac{K \frac{s+1}{s^2+5s+8}}{1 - \left[-K \frac{s+1}{s^2+5s+8} - \frac{s+1}{s^2+5s+8} K_1 \right]}$$

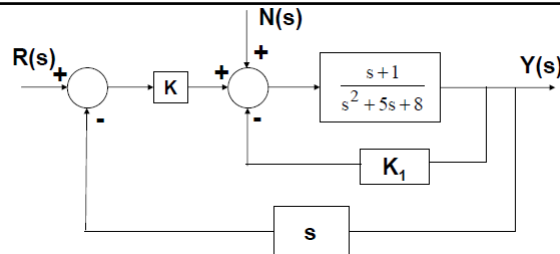
$$H(s) = \frac{Y(s)}{R(s)} = \frac{K \frac{s+1}{s^2+5s+8}}{1 - \left[-K \frac{s+1}{s^2+5s+8} - \frac{s+1}{s^2+5s+8} K_1 \right]}$$

$$H(s) = \frac{Y(s)}{R(s)} = \frac{K \frac{s+1}{s^2+5s+8}}{1 + K \frac{s+1}{s^2+5s+8} + \frac{s+1}{s^2+5s+8} K_1}$$

$$H(s) = \frac{Y(s)}{R(s)} = \frac{Ks + K}{s^2 + 5s + 8 + Ks + K + K_1s + K_1}$$

$$H(s) = \frac{Y(s)}{R(s)} = \frac{Ks + K}{s^2 + (5 + K + K_1)s + (8 + K + K_1)}$$

Example



$$Y(s) = \frac{K \frac{s+1}{s^2+5s+8} R(s) + \frac{s+1}{s^2+5s+8} N(s)}{1 - \left[-K \frac{s+1}{s^2+5s+8} s - \frac{s+1}{s^2+5s+8} K_1 \right]}$$

$$Y(s) = \frac{K \frac{s+1}{s^2+5s+8} R(s) + \frac{s+1}{s^2+5s+8} N(s)}{1 - \left[-K \frac{s+1}{s^2+5s+8} s - \frac{s+1}{s^2+5s+8} K_1 \right]}$$

$$Y(s) = \frac{(Ks + K)R(s) + (s+1)N(s)}{1 + Ks^2 + Ks + K_1s + K_1}$$

$$Y(s) = \underbrace{\frac{Ks + K}{1 + Ks^2 + (K + K_1)s + K_1}}_{\text{With respect to } R(s)} R(s) + \underbrace{\frac{s+1}{1 + Ks^2 + (K + K_1)s + K}}_{\text{With respect to } N(s)} N(s)$$