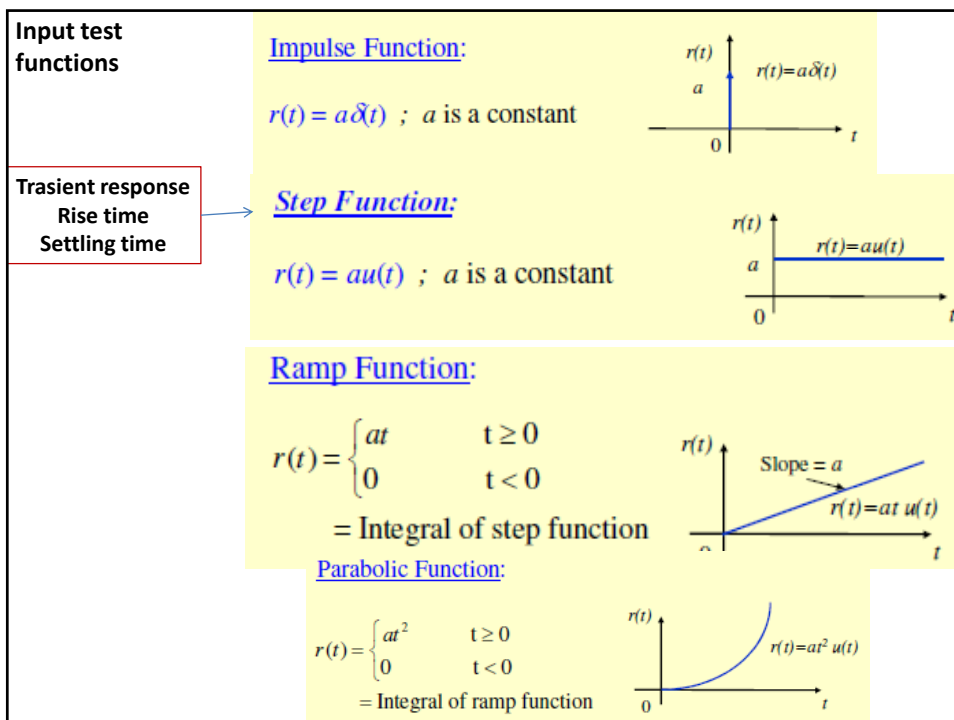


EEM322 : Lecture 5
Transfer function: Zeros; poles
TIME RESPONSE
 First order system
 Second order systems
 Transient response
 Rise time
 Settling time
 Steady state



Time response:**Poles, Zeros, and System Response:**

A differential equation or taking the inverse Laplace transform, enable us to evaluate this output response, these techniques are laborious and time-consuming.

The use of poles and zeros and their relationship to the time response of a system is easy technique:

Poles of a Transfer Function

The poles of a transfer function are the values of the Laplace transform variable, s , that cause the transfer function to become infinite or any roots of the denominator of the transfer function that are common to roots of the numerator.

poles determine the nature of the time response:

Poles of the input function determine the form of the **forced response**, and poles of the transfer function determine the form of the natural response

zeros of a transfer function

The zeros of a transfer function are (1) the values of the Laplace transform variable, s , that cause the transfer function to become zero, or

Transfer Fonksiyonu:

Başlangıç koşulları sıfır kabul edilerek bir sistemin cevap fonksiyonu (çıkışı) ile sürücü fonksiyonu (giriş) arasındaki Laplas transformasyonları oranına transfer fonksiyonu denir.

Transfer fonksiyonu sistemin dinamik karakteristiklerini tanımlar. Sistem özelliğidir. Sistemin fiziksel yapısı hakkında bilgi vermez, farklı fiziksel sistemlerin transfer fonksiyonları aynı olabilir.

Kutuplar:

Transfer fonksiyonunun paydasının kökleridir. Kompleks kutuplar tüm sistemin enerji depolama karakteristikleri ile alakalı doğal frekanslarını temsil eder.

Kompleks kutuplar sistemin doğal frekanslarıdır, Kompleks kutuplar sistemin değişik iç enerji depolama elamanları arasında enerjinin serbestçe dolaştığı doğal frekansları temsil eder.

Sıfırlar:

Transfer fonksiyonunun payının kökleridir. Example:

Sıfırlar:-1, -15

Kutuplar:0, -3, -5, -8 (çift)

$$G(s) = \frac{K(s+1)(s+15)}{s(s+3)(s+5)(s+8)^2}$$

Poles, Zeros, and System Response (Example)

Örnek: $C(s) = \frac{K_1}{s} + \frac{K_2}{(s+2)} + \frac{K_3}{(s+4)} + \frac{K_4}{(s+5)}$

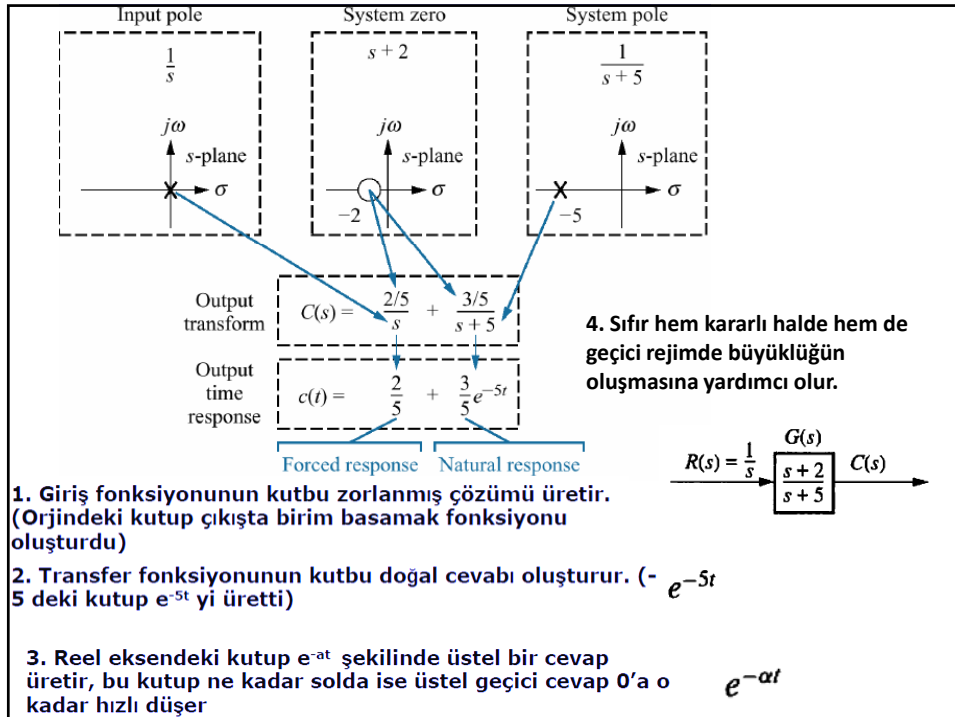
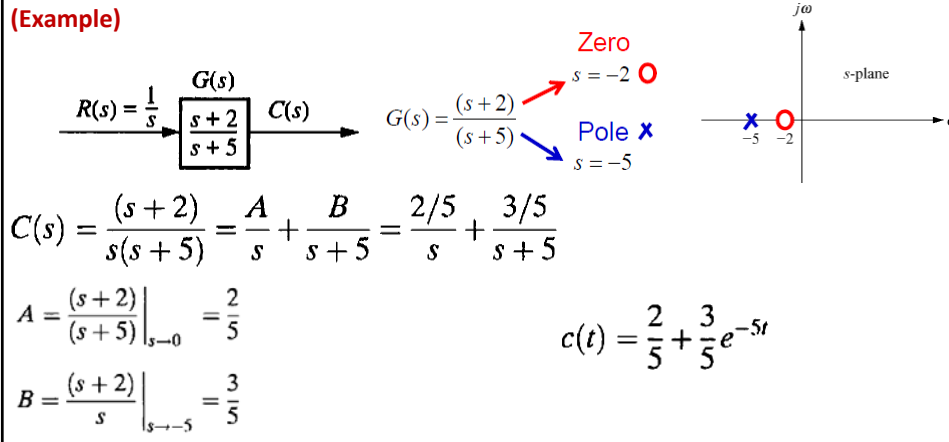
Sisteminin cevabı:

$$c(t) = K_1 + K_2 e^{-2t} + K_3 e^{-4t} + K_4 e^{-5t}$$

Zorlanmış Çözüm

Doğal Çözüm

(Example)



Evaluating Response Using Poles:

The technique of using poles to obtain the form of the system response. We will learn to write the form of the response by inspection. **Each pole of the system transfer function that is on the real axis generates an exponential response that is a component of the natural response.** The input pole generates the forced response.

Example: each system pole generates an exponential as part of the natural response. The input's pole generates the **forced response**. Thus

$$R(s) = \frac{1}{s} \rightarrow \boxed{\frac{(s+3)}{(s+2)(s+4)(s+5)}} \rightarrow C(s) \quad C(s) \equiv \underbrace{\frac{K_1}{s}}_{\text{Forced response}} + \underbrace{\frac{K_2}{s+2} + \frac{K_3}{s+4} + \frac{K_4}{s+5}}_{\text{Natural response}}$$

Taking the inverse Laplace transform,

$$c(t) \equiv \underbrace{K_1}_{\text{Forced response}} + \underbrace{K_2 e^{-2t} + K_3 e^{-4t} + K_4 e^{-5t}}_{\text{Natural response}}$$

poles on the real axis generate exponential responses.

PROBLEM: A system has a transfer function:

Write, the output, $c(t)$, in general terms if the input is a **unit step**.

$$G(s) = \frac{10(s+4)(s+6)}{(s+1)(s+7)(s+8)(s+10)}$$

forced response

$$G(s) = \frac{10(s+4)(s+6)}{(s+1)(s+7)(s+8)(s+10)}$$

ANSWER:

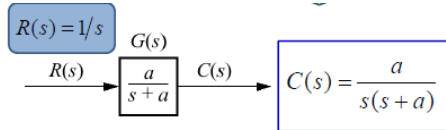
$$c(t) \equiv A + B e^{-t} + C e^{-7t} + D e^{-8t} + E e^{-10t}$$

poles determine the nature of the time response: Poles of the input function determine the form of the forced response, and poles of the transfer function determine the form of the natural response.

FIRST-ORDER SYSTEM

First-order system =
without zero:

$$G(s) = \frac{a}{(s+a)}$$



$$c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$$

First-order system response to a unit step:

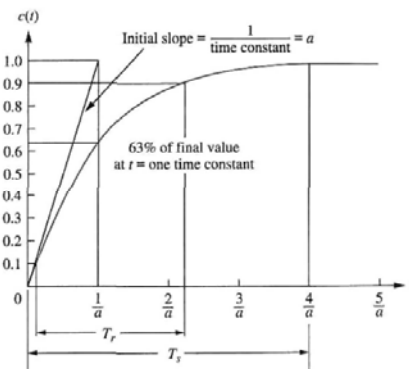
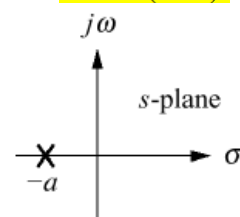
The input pole at the origin generated the forced response $C_f(t) = 1$, and the Sys. pole at $-a$, as shown in Figure, generated the natural response

$$c_n(t) = -e^{-at}$$

Let us examine the significance of parameter a , the only parameter needed to describe the transient response. When $t = 1/a$

$$e^{-at}|_{t=1/a} = e^{-1} = 0.37$$

$$c(t)|_{t=1/a} = 1 - e^{-at}|_{t=1/a} = 1 - 0.37 = 0.63$$

**Time Constant**

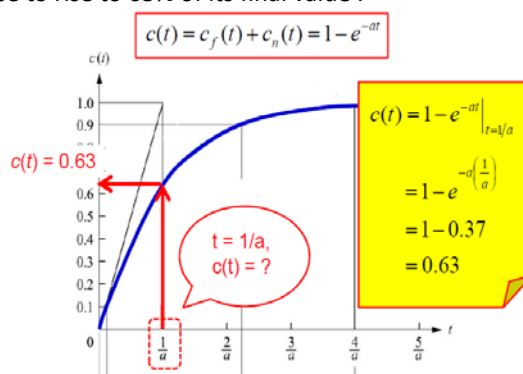
We call $1/a$ the **time constant of the response**. The time constant can be described as the time for e^{-at} to decay to 37% of its initial value. Alternately, the time constant is the time it takes for the step response to rise to 63% of its final value.

Rise Time, T_r : Rise time is defined as the time for the **waveform to go from 0.1 to 0.9 of its final value**.

Rise time is found by solving Eq. for the difference in time at $c(t) = 0.9$ and $c(t) = 0.1$.

$$c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$$

$$T_r = \frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a}$$



Settling Time, T_s : Settling time is defined as the time for the response to reach, and stay within, 2% of its final value. Letting $c(t) = 0.98$ in Eq. and solving for time, t , we find the settling time to be

$$T_s = \frac{4}{a}$$

$$c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$$

FIRST-ORDER SYSTEM: Rise time: Time for the waveform to go from 0.1 to 0.9 of its final value.

Step response:

$$c(t) = 1 - e^{-at}$$

$$t_r = \frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a}$$

When $c(t) = 0.1$:

$$0.1 = 1 - e^{-at}$$

$$e^{-at} = 0.9$$

$$-at = \ln 0.9 = -0.105$$

$$t \approx \frac{0.11}{a}$$

When $c(t) = 0.9$:

$$0.9 = 1 - e^{-at}$$

$$e^{-at} = 0.1$$

$$-at = \ln 0.1 = -2.303$$

$$t \approx \frac{2.31}{a}$$

Settling time:
time for response to reach,
and stay within, 2% of its
final value.

Step response:

$$c(t) = 1 - e^{-at}$$

Letting $c(t) = 0.98$:

$$0.98 = 1 - e^{-at}$$

$$e^{-at} = 0.02$$

$$-at = \ln 0.02 = -3.912$$

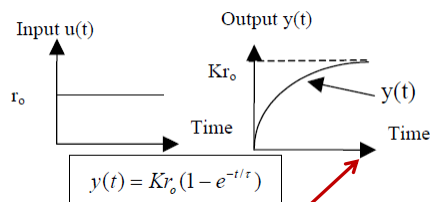
$$t \approx \frac{4}{a}$$

$$t_s = \frac{4}{a}$$

Consider a simple first-order system
Find the time, constant and value K

$$G(s) = \frac{K}{(s + a)}$$

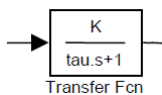
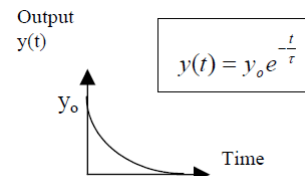
FIRST-ORDER SYSTEM



$$y(t) = y_{\text{forced}}(t) + y_{\text{free}}(t)$$

Zorlanmış Çözüm

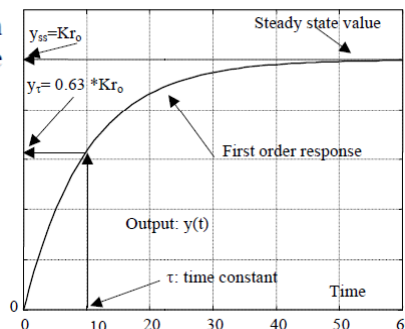
Doğal Çözüm



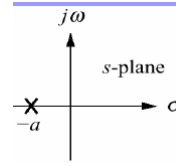
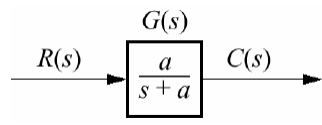
K is referred to as the **system gain** and τ is called the **time constant** for the system.

$$G(s) = \frac{K}{\tau s + 1} \quad \text{Pole-zero form}$$

$$G_{pz}(s) = \frac{K_{pz}}{s + a} \quad K_{pz} = K/\tau, a = 1/\tau$$



Birinci Derece Sistem Cevabı ve Özellikleri



Prof.Dr.Galip Cansever
Ders notları

Eğer $R(s)=1/s$ birim basamak ise $C(s) = \frac{a}{s(s+a)}$

$$c(t) = 1 - e^{-at}$$

a 'ya göre sistemi inceleyelim;

$$t = \frac{1}{a} \Rightarrow e^{-at} \Big|_{t=\frac{1}{a}} = e^{-1} = 0.37$$

$$x(t) = 1 - e^{-at} \Big|_{t=\frac{1}{a}} = 1 - 0.37 = 0.63$$

The **time constant associated with this system is $\tau=1/a$** . **The time constant tells how quickly the system responds.** For example, **if $a=1$, then the system responds on the order of $\tau=1$ second**; however, if **$a=100$** , then the system responds on the order of **$\tau=0.01$ seconds**. So, systems that respond quickly have large values of a , and systems that respond slowly have small values of a .

Yükselme Zamanı, T_r :

Yükselme zamanı cevabın %10'nun dan %90'nına ulaşınca kadar geçen süre olarak tanımlanır.

$$c(t) = 1 - e^{-at}$$

Denkleminde $c(t)=0.9$ ve $c(t)=0.1$ zamanlarında fark alıncak olursa

$$0.9 = 1 - e^{-at} \Rightarrow -t_2 = \frac{\ln(0.1)}{a}$$

$$0.1 = 1 - e^{-at} \Rightarrow -t_1 = \frac{\ln(0.9)}{a}$$

$$T_r = t_2 - t_1 = \frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a} \quad \text{olarak bulunur.}$$

Yerleşme Zamanı, T_s :

Yerleşme zamanı cevabın %98'ine ulaşınca kadar geçen süre olarak tanımlanır.

$$c(t) = 1 - e^{-at}$$

Denkleminde $c(t)=0.98$ olarak alınırsa

$$0.98 = 1 - e^{-at} \Rightarrow -t = \frac{\ln(0.02)}{a} \quad T_s = \frac{4}{a} \quad \text{olarak bulunur.}$$

Second-Order Systems: Introduction:

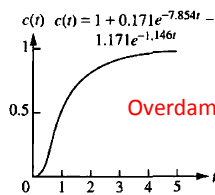
varying a first-order system's parameter simply changes the speed of the response,

A second-order system exhibits a wide range of responses,

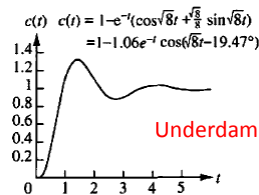
changes in the parameters of a second-order system can change the form of

the response. like a first-order system, or, depending on component values, display damped or pure oscillations for its transient response.

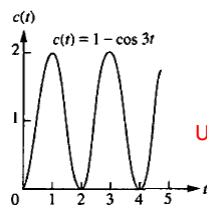
SECOND-ORDER SYSTEMS: Overdamped-response; Underdamped-response; Critically damped, Undamped



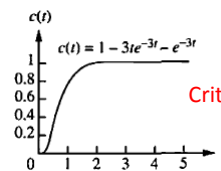
Overdamped-



Underdamped



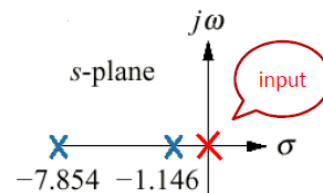
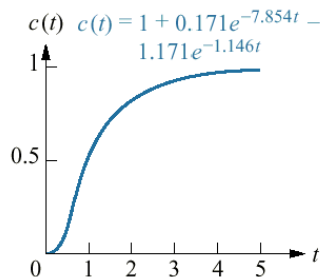
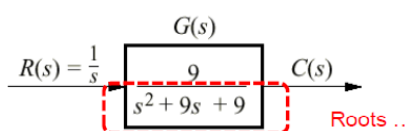
Undamped



Critically damped

SECOND-ORDER SYSTEMS: Overdamped-response:

Aşırı Sönümlü: İki kök reel eksen üzerindeyken oluşan cevaptır.



Poles:

$$-\sigma_1, -\sigma_2$$

Natural response:

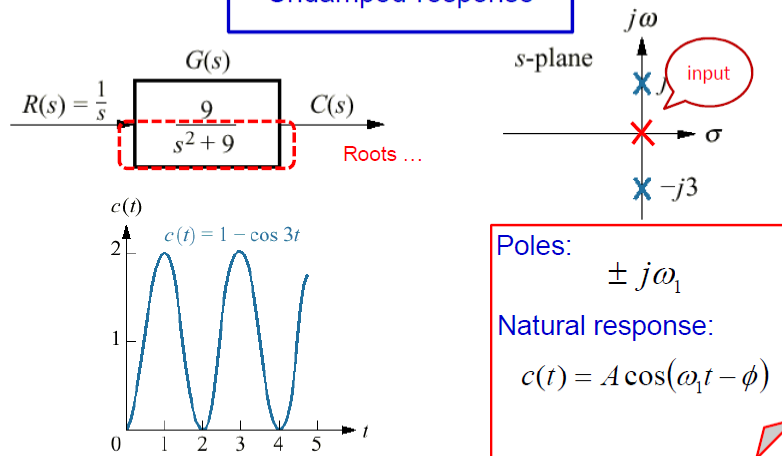
$$c(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$$

$$c(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$$

SECOND-ORDER SYSTEMS.

Osilasyonlu Cevap: İki kök imajiner eksen üzerindeyken oluşan cevabdır.

Undamped-response



If there were no damping, we would have a pure sinusoidal response. Thus, the poles would be on the imaginary axis and the TF would have the form,

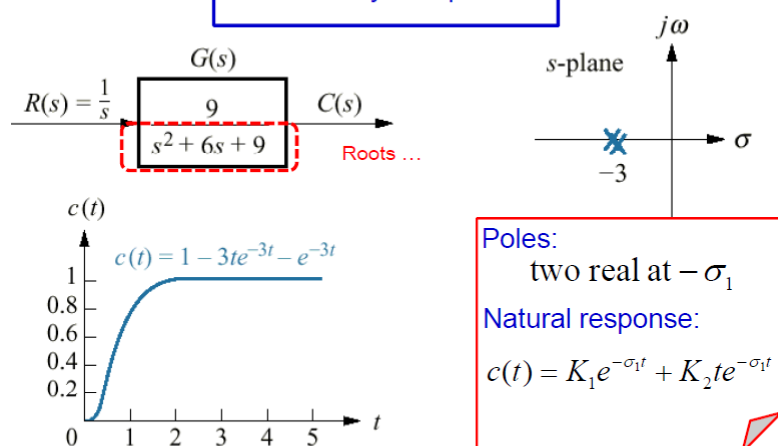
$$c(t) = A \cos(\omega_1 t - \phi)$$

$$b = \omega_n^2$$

SECOND-ORDER SYSTEMS

$$c(t) = K_1 e^{-\sigma_1 t} + K_2 t e^{-\sigma_1 t}$$

Critically damped

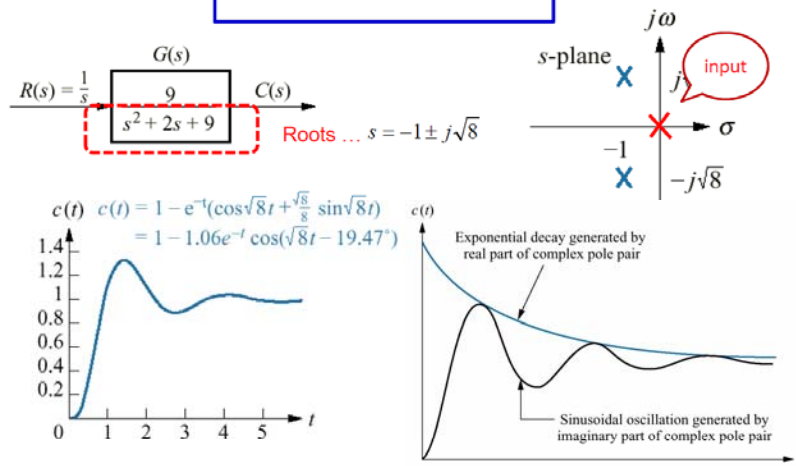


Kritik Sönümlü Cevap: İki katlı kök reel eksen(negatif bölge) üzerindeyken oluşan cevabdır.

damped. Critically damped responses are the fastest possible without the overshoot that is characteristic of the underdamped response

SECOND-ORDER SYSTEMS

Underdamped-response



Sönümlü: Kompleks eşlenik kökler varken olan cevabdır
 Kutbun reel kısmı sinüzoidalın genliğinin üstel düşüm frekansına denk gelirken, **imajiner kısmı ise sinüzoidalın osilasyon frekansına karşılık gelir.**
 Sinüzoidal'ın frekansına **sönümlü osilasyon frekansı** denir, ω_d

Poles:

$$-\sigma_d \pm j\omega_d$$

Natural response:

$$c(t) = Ae^{-\sigma_d t} \cos(\omega_d t - \phi)$$

Aşırı Sönümlü:

Kökler: $-\sigma_1, -\sigma_2$

Özet

$$c(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$$

$$c(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$$

Sönümlü:

Kökler: $-\sigma_d \pm j\omega_d$

$$c(t) = Ae^{-\sigma_d t} \cos(\omega_d t - \phi)$$

Osilasyonlu Cevap:

Kökler: $\pm j\omega_1$

$$c(t) = A \cos(\omega_1 t - \phi)$$

Kritik Sönümlü Cevap:

Kökler: $-\sigma_1, -\sigma_1$

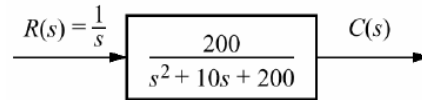
$$c(t) = K_1 e^{-\sigma_1 t} + K_2 t e^{-\sigma_1 t}$$

Category	Poles	$c(t)$
Overdamped	Two real: $-\sigma_1, -\sigma_2$	$K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$
Underdamped	Two complex: $-\sigma_d \pm j\omega_d$	$Ae^{-\sigma_d t} \cos(\omega_d t - \phi)$
Undamped	Two imaginary: $\pm j\omega_n$	$A \cos(\omega_n t - \phi)$
Critically damped	Repeated real: $-\sigma_d$	$K_1 e^{-\sigma_d t} + K_2 t e^{-\sigma_d t}$

Example:

Form of **Underdamped Response** Using Poles PROBLEM: By inspection, write the form of the step response of the system in Figure

$$s = -5 \pm j13.23$$



The **real part, -5**, is the exponential frequency for the damping. It is also the reciprocal of the time constant of the decay of the oscillations. The **imaginary part, 13.23**, is the radian frequency for the sinusoidal oscillations.

Sisteminin birim basamak cevabını yazınız.

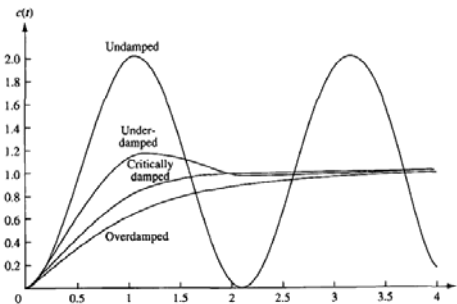
Eşlenik kutuplar: $s_{1,2} = -5 \pm j13.23$

$$c(t) = K_1 + e^{-5t} (K_2 \cos(13.23t) + K_3 \sin(13.23t))$$

$$c(t) = K_1 + K_4 e^{-5t} (\cos(13.23t) - \phi)$$

$$\phi = \tan^{-1} \left(\frac{K_3}{K_2} \right) \quad K_4 = \sqrt{K_2^2 + K_3^2}$$

Examples: For each of the following transfer functions, write, by inspection, the general form of the step response:

**Example:**

$$(a) \quad G(s) = \frac{400}{s^2 + 12s + 400}$$

$$(b) \quad G(s) = \frac{900}{s^2 + 90s + 900}$$

$$(c) \quad G(s) = \frac{225}{s^2 + 30s + 225}$$

$$(d) \quad G(s) = \frac{625}{s^2 + 625}$$

Answers

$$(a) \quad c(t) = A + B e^{-6t} \cos(19.08t + \phi)$$

$$(b) \quad c(t) = A + B e^{-78.54t} + C e^{-11.46t}$$

$$(c) \quad c(t) = A + B e^{-15t} + C t e^{-15t}$$

$$(d) \quad c(t) = A + B \cos(25t + \phi)$$

The General Second-Order System:

we define two physically meaningful specifications for second-order systems. These quantities can be used to describe the **characteristics** of the second-order transient response just as **time constants describe the first-order** system response. The two quantities are **called natural frequency and damping ratio**
Natural frequency

Doğal Frekans, ω_n : İkinci derece bir sistemin doğal frekansı sistemin sönümsüz osilasyon frekansıdır.

For example, the frequency of oscillation of a series *RLC* circuit with the **resistance shorted** would be the natural frequency.

Sönüm oranı, ζ : Üstel düşüm frekansının doğal frekansa oranıdır.

$$\zeta = \text{Üstel düşüm frekansı} / \text{Doğal frekans (rad/s)}$$

$$\zeta = (1/2\pi) (\text{Doğal periyot} / \text{Üstel zaman sabiti})$$

Damping Ratio, ζ : This measures the amount of damping. For underdamped systems ζ lies in the range $[0, 1]$:

THE GENERAL SECOND-ORDER SYSTEM

The quantities used to describe the characteristics of the second-order transient response:

□ **Natural frequency, ω_n**

- frequency of oscillation of the system without damping.

□ **Damping ratio, ζ**

$$\begin{aligned} \zeta &= \frac{\text{Exponential decay frequency}}{\text{Natural frequency (rad/sec)}} \\ &= \frac{1}{2\pi} \frac{\text{Natural period (seconds)}}{\text{Exponential time constant}} = \frac{|\sigma_d|}{\omega_n} \end{aligned}$$

Consider a general system

$$G(s) = \frac{b}{s^2 + as + b}$$

The exponential decay frequency σ_d is the real-axis component of the poles of a critically damped or underdamped system.

Natural frequency, ω_n is the frequency of oscillation of the system.

Pole on the $j\omega$ -axis at $\pm j\sqrt{b}$,

$$\omega_n = \sqrt{b} \quad \Rightarrow \quad b = \omega_n^2$$

Assuming an underdamped system, the complex poles have a real part, ω_n equal to $-a/2$.

$$\zeta = \frac{\text{Exponential decay frequency}}{\text{Natural frequency (rad/sec)}} = \frac{|\sigma|}{\omega_n} = \frac{a/2}{\omega_n}$$

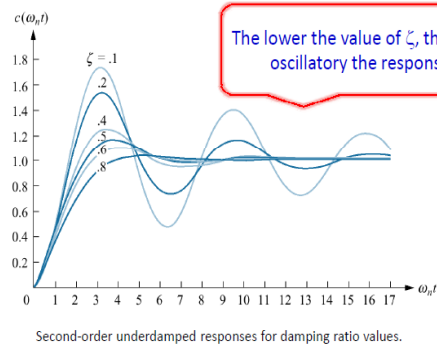
THE GENERAL SECOND-ORDER SYSTEM

$$a = 2\zeta\omega_n$$

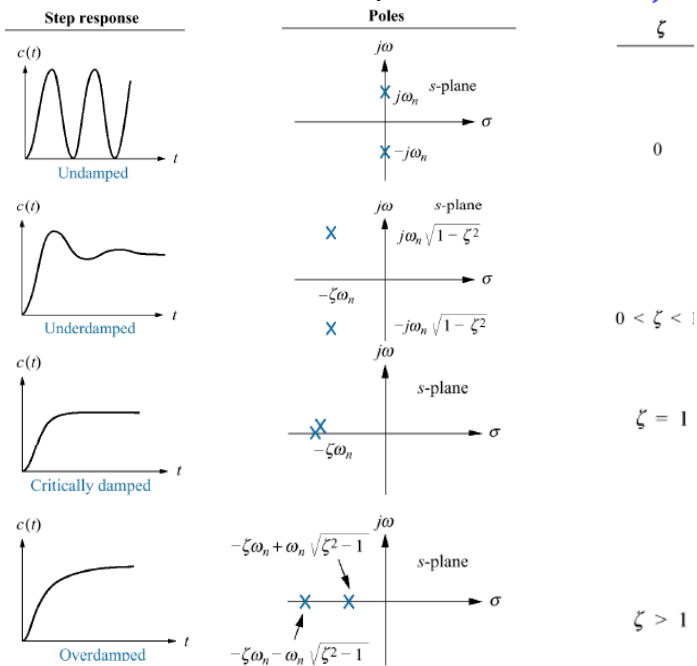
General second-order transfer function:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$



The various cases of second-order response are a function of ζ ;



Örnek:

$$G(s) = \frac{36}{s^2 + 4.2s + 36} \quad \text{ise}$$

$$2\zeta\omega_n = 4.2 \quad \text{ve} \quad \omega_n^2 = 36$$

$$\omega_n = 6 \quad \zeta = \frac{4.2}{2 \times 6} = 0.35$$

Genel olarak:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

İkinci derece sistemin kökleri:

$$s_{1,2} = -\zeta\omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$

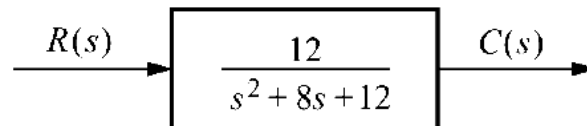
Example :

Given the transfer function as below

find ζ and ω_n

$$G(s) = \frac{36}{s^2 + 4.2s + 36} \quad \omega_n^2 = 36 \quad 2\zeta\omega_n = 4.2$$

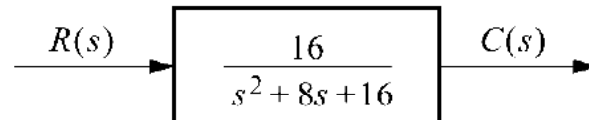
$$\omega_n = \sqrt{36} = 6 \quad \zeta = \frac{4.2}{2 \times \omega_n} = \underline{\underline{0.35}}$$

Örnek:

Sisteminin ne tür cevabı olacağını bulunuz.

$$a = 2\zeta\omega_n \quad \text{ve} \quad \omega_n = \sqrt{b} \Rightarrow \zeta = \frac{a}{2\sqrt{b}} = \frac{8}{2\sqrt{12}}$$

$$\zeta = 1.155 \quad \zeta > 1 \quad \text{Olduğu için aşırı sönümlüdür}$$

Örnek:

Sisteminin ne tür cevabı olacağını bulunuz.

$$\zeta = \frac{8}{2\sqrt{16}} \Rightarrow \zeta = 1$$

$$\zeta = 1 \quad \text{Olduğu için kritik sönümlüdür}$$

Prof.Dr.Galip Cansever
Ders notları

Given the transfer function as below

find ζ and ω_n .

$$G(s) = \frac{36}{s^2 + 4.2s + 36} \quad \omega_n^2 = 36$$

$$\omega_n = \sqrt{36} = 6$$

$$2\zeta\omega_n = 4.2$$

$$\zeta = \frac{4.2}{2 \times \omega_n} = 0.35$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$(a) \quad G(s) = \frac{400}{s^2 + 12s + 400}$$

$\zeta = 0.3, \omega_n = 20$; system is underdamped

$$(b) \quad G(s) = \frac{900}{s^2 + 90s + 900}$$

$\zeta = 1.5, \omega_n = 30$; system is overdamped

$$(c) \quad G(s) = \frac{225}{s^2 + 30s + 225}$$

$\zeta = 1, \omega_n = 15$; system is critically damped

$$(d) \quad G(s) = \frac{625}{s^2 + 625}$$

$\zeta = 0, \omega_n = 25$; system is undamped

UNDERDAMPED SECOND-ORDER SYSTEMS

General second-order systems:

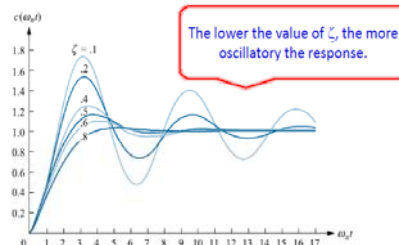
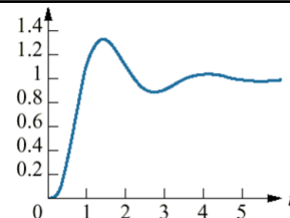
$$C(s) = \frac{1}{s} \times \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Expanding by partial-fraction, yields;

$$C(s) = \frac{1}{s} - \frac{(s + \zeta\omega_n) + \frac{\zeta}{\sqrt{1-\zeta^2}}\omega_n\sqrt{1-\zeta^2}}{(s + \zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}$$

Taking the inverse Laplace transform,

$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cos(\omega_n \sqrt{1-\zeta^2} t - \phi) \quad \text{where } \phi = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)$$



Other parameters associated with the underdamped response are **rise time, peak time, percent overshoot, and settling time.**

The specifications for under-damped second order systems are defined as follows

Rise time, T_r	The time required for the waveform to go from 0.1 of the final value to 0.9 of the final value.
Peak time, T_p	The time required to reach the first, or maximum, peak.
Percent overshoot, %OS	The amount that the waveform overshoots the steady-state, or final, value at the peak time , expressed as a percentage of the steady-state value.
Settling time, T_s	The time required for the transient's damped oscillations to reach and stay within $\pm 2\%$ of the steady-state value.

1. Tepe Süresi, T_p :

Sistem cevabının tepe veya maksimum noktaya ulaştığında geçen süre.

2. Aşım, %OS:

Sistem cevabının tepe veya maksimum noktası ile kararlı haldeki değeri arasındaki farkın kararlı haldeki değere oranıdır. % olarak ifade edilir.

3. Yükselme Zamanı, T_r :

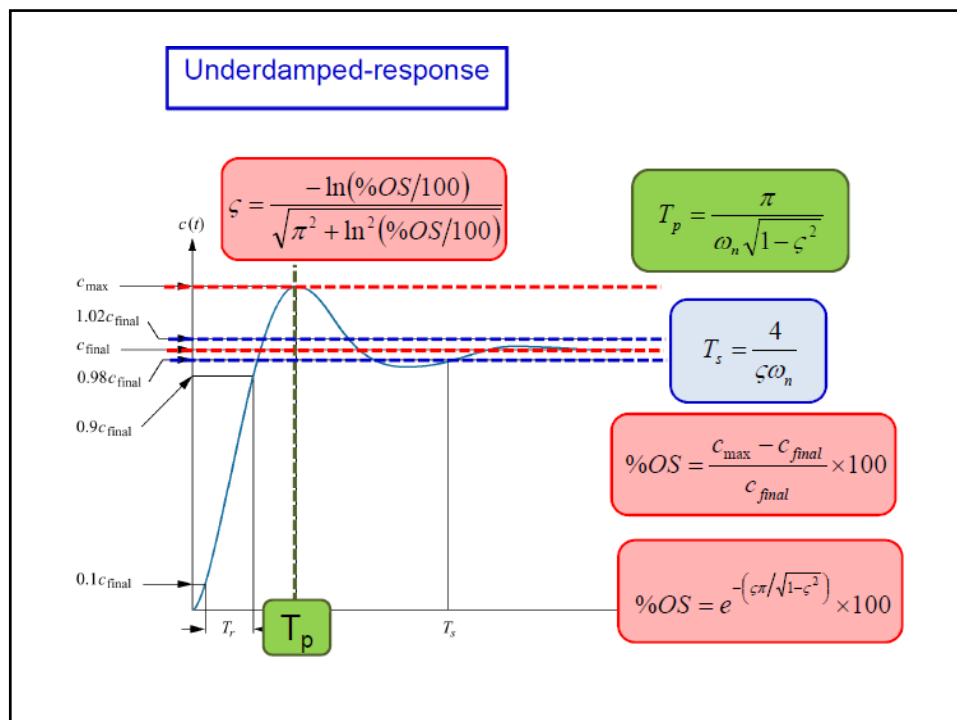
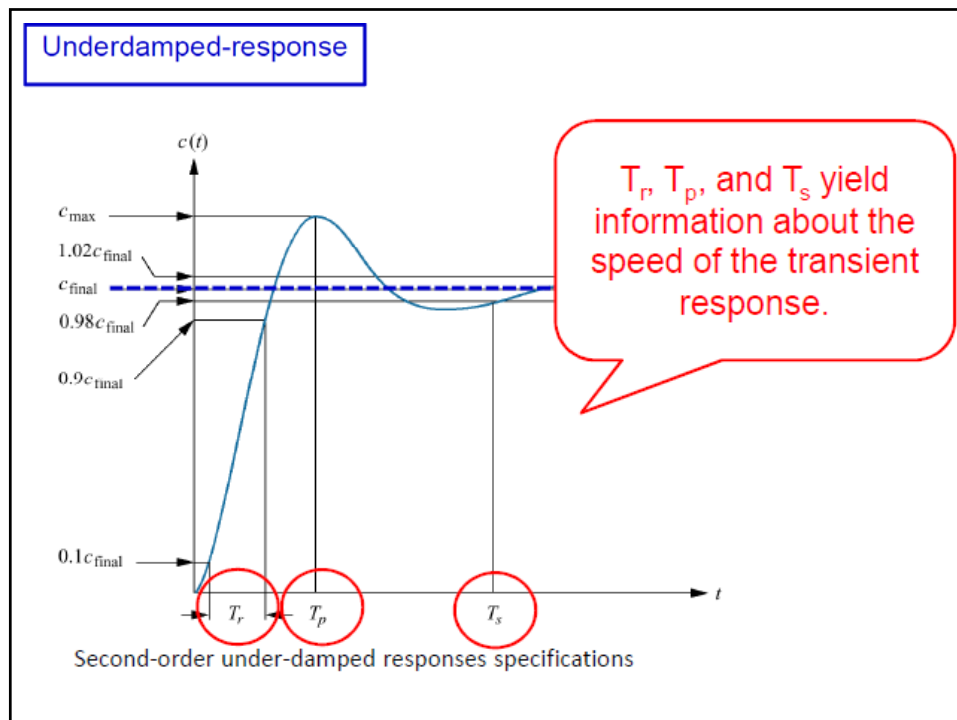
Sistem cevabının %10'nun dan %90'nına ulaşmaya kadar geçen süre olarak tanımlanır.

4. Yerleşme Zamanı, T_s :

Sistem cevabının %98'ine ulaşmaya kadar geçen süre olarak tanımlanır.

Bu tip bilgiler tasarımcının cevabın hızının veya doğasının sistem performansını azaltıp azaltmadığına karar vermesi açısından önemlidir.

Örneğin bir CD okuyucusunun kafasının bilgiyi okumak için kararlı hale gelmesi tüm bilgisayar performansını etkiler.



Evaluation of T_p

T_p is found by differentiating $c(t)$ in Eq. and finding the first zero crossing after $t = 0$. This task is simplified by "differentiating" in the frequency domain

$$c(t) = 1 - e^{-\zeta\omega_n t} \left(\cos \omega_n \sqrt{1 - \zeta^2} t + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin \omega_n \sqrt{1 - \zeta^2} t \right)$$

$$\mathcal{L}[\dot{c}(t)] = sC(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\mathcal{L}[\dot{c}(t)] = \frac{\omega_n^2}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)} = \frac{\frac{\omega_n}{\sqrt{1 - \zeta^2}} \omega_n \sqrt{1 - \zeta^2}}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)}$$

$$\dot{c}(t) = \frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1 - \zeta^2} t$$

Setting the derivative equal to zero yields

$$\omega_n \sqrt{1 - \zeta^2} t = n\pi \longrightarrow t = \frac{n\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

n nin her bir değeri yerel maksimum veya minimumu gösterir. $n=0$, $t=0$ anına karşılık gelir ve eğim sıfırdır. n nin değeri 1 olması birinci tepe zamanına karşılık gelir. Böylece:

$$T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

Aşım, %OS, nin incelenmesi

$$\%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1 - \zeta^2}}\right)} \times 100$$

$$\%OS = \frac{c_{\max} - c_{\text{final}}}{c_{\text{final}}} \times 100$$

$$\zeta = \frac{-\ln(\%OS / 100)}{\sqrt{\pi^2 + \ln^2(\%OS / 100)}}$$

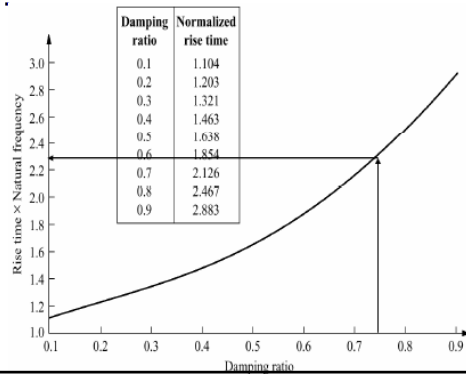
Yerleşme Süresi, T_s , nin incelenmesi

Yerleşme süresini bulabilmek için $c(t)$ nin kararlı hal c_{final} değerini %98 sine ulaştığı zamanı hesaplamamız gerekir. Tanımdan hatırlanacağı üzere yerleşme süresi azalan sinüsoidalın genliğinin 0.02' ye ulaşma süresidir. Dolayısıyla:

$$T_s = \frac{4}{\zeta\omega_n}$$

Yükselme Süresi, T_r , nin incelenmesi

Yükselme zamanı ve sönüm arasındaki ilişki aşağıdaki şekil ile bulunabilir.



Örnek: $G(s) = \frac{100}{s^2 + 15s + 100}$ Sistemi için, T_p , %OS ve T_s 'yi bulunuz

$$a = 2\zeta\omega_n \quad \omega_n = \sqrt{b} = \sqrt{100} = 10 \Rightarrow \zeta = \frac{a}{2\sqrt{b}} = \frac{15}{2\sqrt{100}} = 0.75$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{10\sqrt{1-0.75^2}} = 0.475$$

$$\%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100 = e^{-\left(\frac{0.75\pi}{\sqrt{1-0.75^2}}\right)} \times 100 = 2.838$$

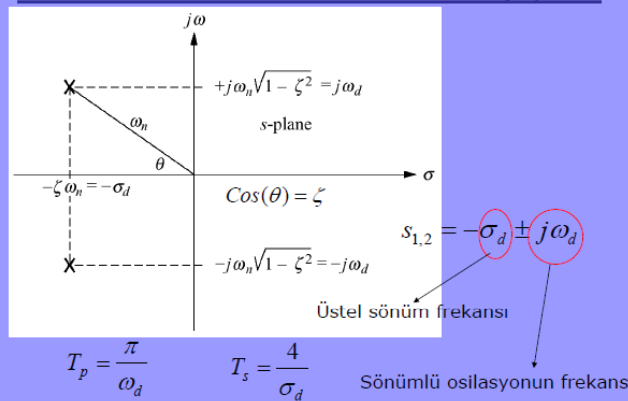
$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.75 \times 10} = 0.533$$

Şekilden
 $T_r = \frac{2.3}{10} = 0.23$

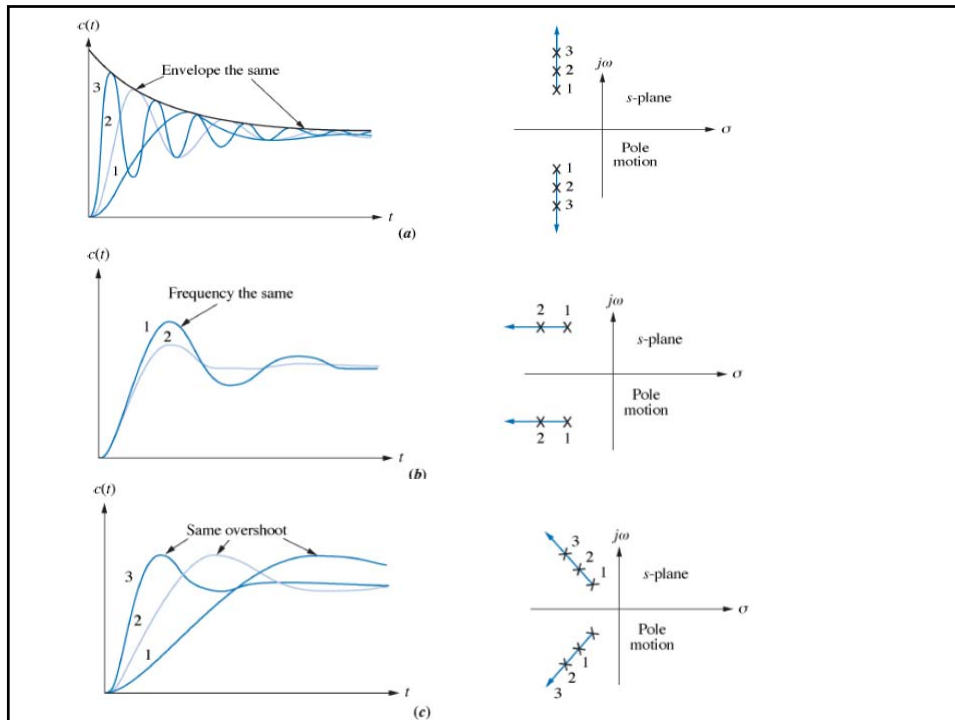
Location of the poles:

Relate peak time, percent overshoot, and settling time to the natural frequency and the damping ratio. Now let us relate these quantities to the location of the poles that generate these characteristics.

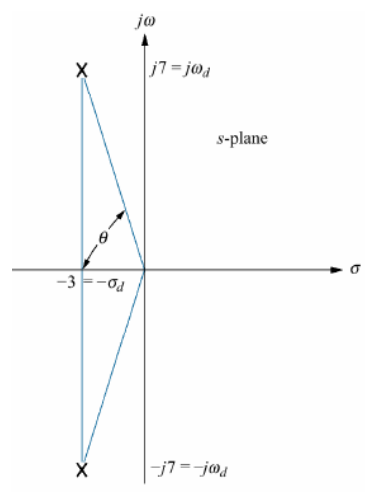
Sönümlü İkinci Derece Sistemlerin Kutup Çizimi



radial distance from the origin to the pole is the natural frequency, ω_n , and the $\cos \theta = \zeta$.

**Örnek:**

Kutup şekli verilen Sistem için, ζ , ω_n , T_p , %OS ve T_s 'yi bulunuz



$$\zeta = \cos(\theta)$$

$$= \cos(\arctan(7/3)) = 0.394$$

$$\omega_n = \sqrt{7^2 + 3^2} = 7.616 \text{ rad/s}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{7} = 0.449 \text{ s}$$

$$\%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$$

$$= e^{-\left(\frac{0.394\pi}{\sqrt{1-0.394^2}}\right)} \times 100 = \%26.018$$

$$T_s = \frac{4}{\sigma_d} = \frac{4}{3} = 1.33 \text{ s}$$